Abstract

Deficit finance, a.k.a. pay-go policy, is free when growth rates routinely exceed safe government borrowing rates. Or so many say. This note presents four counterexamples based on four versions of a simple OLG economy. In each version the growth rate exceeds the safe rate for one of four reasons – uninsured idiosyncratic risk, uninsured aggregate risk, policy uncertainty, and imperfect financial intermediation. Deficit finance does not directly address any of these problems. What works, respectively speaking, is progressive taxation, bilateral intergenerational risk-sharing, early policy resolution, and improved intermediation. The four examples thus show that seemingly free deficits may be more costly than they appear. Indeed, inefficient pay-go policy can even lower the government’s borrowing rate, encouraging yet more deficit finance.
1 Introduction

Some suggest fiscal deficits may have no cost when public borrowing rates average less than economic growth rates.¹ This note demurs. It presents four variants of a simple, two-period, zero-growth, OLG model, each of which features a growth premium – a growth rate that exceeds the safe rate. None provides support for deficit finance, the essence of which is taking from the young to give to the old, henceforth, pay-go policy. In the first model, the negative safe rate reflects idiosyncratic return risk. In the second, there is aggregate return risk. The third features both aggregate risk and policy uncertainty. And, in the fourth, aggregate risk plus imperfect intermediation drives the safe lending rate (i.e., the government borrowing rate) below the growth rate, while leaving the safe private borrowing rate above the growth rate. Model 1’s risk can be dispelled via progressive taxation. Model 2’s risk can be mitigated via bilateral intergenerational risk-sharing. Model 3’s risk can be reduced by constitutional or other policy-commitment mechanisms. Model 4’s dispersion in safe rates can be ameliorated via improved intermediation with, for example, the help of government loan verification, collection, and enforcement. In several of our model variants, pay-go policy can potentially Pareto improve and may be worth considering if targeted policies are politically or otherwise unavailable.

The message of our missives is clear. Judging the welfare impact of deficit finance when interest rates are low is not as simple as it seems. Such policies, while mitigating risk, albeit inefficiently, may benefit current generations at the expense of future generations. They may also produce policy risk, which, paradoxically, lowers the government’s borrowing rate, encouraging yet more deficit finance that further reduces the welfare of future generations. And they may redistribute from borrowers to lenders, where the former face safe rates above and the latter safe rates below the growth rate. Our four parables all show that a low government borrowing rate, in and of itself, does not justify deficit finance. Of course, there may be other reasons for such policies, not considered here, that merit consideration. Our paper simply warns against taking low interest rates as sufficient ground for running deficits.

Our four counter-examples, presented in Section 4 to 7, connect to various strands of literature. Our first model echoes Blanchard and Weil (2001). That paper teaches, in part, that idiosyncratic risk can generate negative real interest rates and illustrates that pay-go policy bears no necessary connection to the source of low rates. Reis (2021), Brunnermeier et al. (2021), and Aguiar et al. (2021) are more recent papers examining the role of idiosyncratic risk in generating low interest rates and the feasibility of Ponzi schemes. Their frameworks, however, are infinite horizon and, thus, circumvent the fundamental concern about pay-go policy – harming young and future generations to benefit the current old. Imrohoroglu et al. (1995) and Conesa and Krueger (1999) show that a pay-go social security system does not necessarily constitute an efficient tool for intragenerational risk-sharing. Our second model is most closely related to Brumm et al. (2021), which traces Blanchard (2019)’s case for deficit finance not to intergenerational redistribution, but to intergenerational and international risk-sharing as well as implicit beggar-thy-neighbor policy. In our third model, as in Phillips et al. (2013), the government itself causes intergenerational risk in running pay-go. Caliendo et al. (2019) explore this idea in more detail for the case of social security reform. As for our fourth model, which builds on Brumm et al. (2020) in stressing the importance of borrowing-lending gaps, its earliest

¹See Blanchard et al. (2020), Summers and Rachel (2019), Blanchard and Summers (2019), and Blanchard (2019) for prominent examples pointing in that direction.
antecedent appears to be Hubbard and Judd (1987). Hurst and Willen (2007) also explore the interplay of borrowing restrictions on social security in a quantitative setting. Finally, Bassetto and Sargent (2020) present an example with ad hoc borrowing constraints that implies, like our model with transaction costs, that increased debt at negative interest rates hurts borrowers and benefits lenders.

2 The Basic Model

Agents live for two periods, working when young and consuming when old. Production is linear in labor and capital. The growth rate of the economy is normalized to zero. The wage equals 1. The return on capital, which fully depreciates each period, is uncertain with expected value \( R \). In particular, each unit of capital returns either \( RH > 1 \) or \( RL < 1 \), with equal probability, where

\[
H = 1 + \theta, \quad L = 1 - \theta. \tag{1}
\]

Expected utility of agents born at \( t \) is

\[
EU_t = \frac{1}{2} C_{t+1,H}^{1-\gamma} + \frac{1}{2} C_{t+1,L}^{1-\gamma}, \tag{2}
\]

where,

\[
C_{t+1,H} = \alpha S + (1 - \alpha)RH, \\
C_{t+1,L} = \alpha S + (1 - \alpha)RL, \tag{3}
\]

and \( \alpha \) is the share of savings invested in the safe bond. The net supply of these bonds is zero. The optimal choice of \( \alpha \) plus the equilibrium condition, \( \alpha = 0 \), imply

\[
\frac{1}{2} (S - RH)(RH)^{-\gamma} + \frac{1}{2} (S - RL)(RL)^{-\gamma} = 0,
\]

which results in

\[
S = \frac{(RH)^{1-\gamma} + (RL)^{1-\gamma}}{(RH)^{-\gamma} + (RL)^{-\gamma}} = RH^{\gamma} + LH^{\gamma} \tag{4}
\]

When there is no risk, there is no risk premium, i.e., when \( H = L = 1, S = R \). The same is true if agents are risk neutral, i.e., when \( \gamma = 0 \). When \( \gamma = 1 \), the logarithmic case, \( S = RHL \).

If \( \theta = 0.9 \), \( HL \) equals 0.19 and \( R \) thus exceeds \( S \) by a factor of more than 5. Suppose \( R = 3 \), then \( S = 0.57 \) — which corresponds, assuming a period length of 30 years, to yearly risky and safe returns of roughly 3.7% and −1.9%, respectively.

3 Running Pay-Go

Given the above model with a safe rate \( S < 1 \), it appears attractive to run a pay-go policy scheme — taking \( T \) from the young each period and giving it to the old — which naturally provides a return of 1 exceeding the safe rate. Note that such a policy corresponds to a debt policy where the government borrows, \( T \), from the young to finance its transfers to the contemporaneous old. When the young are old the government pays a principal, \( T \), plus interest,
(S – 1)T, on its borrowing while taxing them to cover interest. In this setting with negative interest rates the tax will be negative.\footnote{Forced versus voluntary lending by the young to the government may, as Hayashi (1987) showed, make no difference to the equilibrium, even if a portion of the young are borrowing constrained. Our fourth model variant does not satisfy Hayashi’s proposition.} With such a policy in place \( C_{t+1,H} \) and \( C_{t+1,L} \) satisfy

\[
\begin{align*}
C_{t+1,H} &= \alpha(1-T)S + (1-\alpha)(1-T)RH + T, \\
C_{t+1,L} &= \alpha(1-T)S + (1-\alpha)(1-T)RL + T.
\end{align*}
\]

The optimal choice of \( \alpha \) plus the equilibrium condition, \( \alpha = 0 \), now imply

\[
\frac{1}{2}(1-T)(S – RH)((1-T)RH + T)^{-\gamma} + \frac{1}{2}(1-T)(S – RL)((1-T)RL + T)^{-\gamma} = 0,
\]

which implies a safe rate of

\[
S = RH[(1-T)RH + T]^{-\gamma} + L[(1-T)RL + T]^{-\gamma}
\]

\[
\frac{R[T - (1-T)RH]^{-\gamma} + L[T - (1-T)RL]^{-\gamma}}{[(1-T)RH + T]^{-\gamma} + [(1-T)RL + T]^{-\gamma}}.
\]

When \( T = 1 \), i.e. when all resources are used for transfers and none for capital accumulation, the safe rate equals the risky rate \( S = R \). In this case, the safe rate exceeds the pay-go-policy return and the policy is inefficient. However, there is a unique \( T = T^* < 1 \) that results in \( S = 1 \), which follows from the fact that \( S \) is strictly increasing in \( T \), as we now show. After rearranging,

\[
S = RH[(1-T)RH + T]^{-\gamma} + L[RH + T(1-RH)]^{-\gamma}.
\]

Due to \( RL < 1 < RH \) we have

\[
\frac{\partial[RH + T(1-RH)]^{-\gamma}}{\partial T} = \gamma[RH + T(1-RH)]^{-\gamma-1}(1-RH) < 0 \quad \frac{\partial[RH + T(1-RH)]^{-\gamma}}{\partial T}
\]

which implies, given \( L < 1 < H \), that the numerator of \( S \) grows faster than its denominator when \( T \) rises – thus \( S \) increases in \( T \). The impact of an increase in \( T \) at time \( t = 0 \) on expected utility \( EU_t \) of generations born at \( t \geq 0 \) is, using equation (6), given by

\[
\frac{\partial EU_t}{\partial T} = \frac{1}{2}(1-RH)C_{t+1,H}^{-\gamma} + \frac{1}{2}(1-RH)C_{t+1,L}^{-\gamma}
\]

\[
= \frac{1}{2}(1-RH)C_{t+1,H}^{-\gamma} + \frac{1}{2}(1-RH)C_{t+1,L}^{-\gamma}
\]

\[
= \frac{1}{2}(1-S)\left(\frac{1}{2}C_{t+1,H}^{-\gamma} + \frac{1}{2}C_{t+1,L}^{-\gamma}\right),
\]

which is always positive as long as \( S < 1 \). For the initial old, born at \( t = -1 \), the marginal impact of pay-go is

\[
\frac{\partial EU_{-1}}{\partial T} = \frac{1}{2}C_{0,H}^{-\gamma} + \frac{1}{2}C_{0,L}^{-\gamma} > 0.
\]

Hence, if \( S < 1 \), raising \( T \) to \( T^* \), the value of \( T \) at which \( S = 1 \), is the optimal pay-go policy in the sense that it provides each generation (except the initial old) the largest welfare gain of any
pay-go policy. However, were technology non-linear, the policy’s crowding out of capital could, as shown by Blanchard (2019) and Brumm et al. (2021), transform pay-go from win-win to win-lose. Moreover, even when it is Pareto improving, pay-go is not the only Pareto improving policy, let alone the most equitable policy to consider. In the following, we show this for the two extreme cases, when risk is either entirely idiosyncratic or purely aggregate, i.e. when agents’ risky returns are either uncorrelated or perfectly correlated. Our key point is that pursuing a pay-go policy in this context can permanently damage young and future generations insofar as it precludes running more beneficial policy.

4 Idiosyncratic Risk and Progressive Taxation

Consider a variant of our model in which return uncertainty - each agent’s realized value of either $H$ or $L$ – is purely idiosyncratic, i.e. the aggregate economy is deterministic with agents investing in their own risky technologies with uncorrelated returns. Markets are incomplete so that agents cannot insure their idiosyncratic return risk. While pay-go can generate Pareto improvements as shown above, the government could also eliminate idiosyncratic return risk by transferring $\theta R$ from those earning high returns to those earning low returns. Doing so, via progressive taxation, raises $S$ to $R$. It also raises the expected utility of all current and future generations from

$$EU_t = \left(\frac{1}{2}H^{1-\gamma} + \frac{1}{2}L^{1-\gamma}\right)\frac{R^{1-\gamma}}{1-\gamma}$$

(10)

to

$$EU_t = \frac{R^{1-\gamma}}{1-\gamma}.$$  

(11)

Suppose, however, policy makers implemented pay-go rather than intragenerational risk-sharing. Presumably they would set $T$ at the value $T^*$ at which $S = 1$. As shown, relative to no-policy, all generations gain from setting $T$ to $T^*$. But, clearly, pay-go is not the only available Pareto improvement. One indicator of this is that pay-go permanently reduces the economy’s output. To see why, note that the no-policy economy’s output is $1 + R$ per period. Pay-go policy with $T = T^*$ leaves output unchanged at $t = 0$. Thereafter, however, output falls to $1 + (1 - T^*)R$ since setting $T$ to $T^*$ crowds out investment in productive capital. In contrast, progressive taxation keeps output at $1 + R$ forever. Instead of reducing the economy’s productive capacity, it targets the real problem – inefficient risk allocation within cohorts.\(^3\) Once implemented, everyone, including the initial old, consumes $R$ for sure when old. Compared with no-policy, this raises the expected utility of all generations, including that of the initial old. Admittedly, idiosyncratic risk-sharing is not as favorable to the initial old as pay-go. But there is nothing in the problem that justifies singling out the initial old for special treatment. In short, pay-go is a decision to make all current young and future generations worse off relative to progressive taxation. Note, though, that once the $T^*$ policy is implemented, it may be possible

\(^3\) Of course, reducing idiosyncratic risk through progressive taxation may distort, for instance, labor supply, investment in human capital, or entrepreneurial investment. On the other hand, pay-go/deficit finance comes with its own distortions. Absent adverse selection and moral hazard issues, private human capital insurance could obviate the need for progressive taxation.
to gradually move to the first best policy by substituting pay-go policy for progressive tax policy, achieving a Pareto improvement in the process.

5 Aggregate Risk and State-Dependent Pay-Go

Next, assume return uncertainty is purely aggregate, i.e. individual returns are perfectly correlated. In this case, intragenerational redistribution cannot improve allocations, while pay-go policy is as effective as in the case of idiosyncratic risk. However, there is a more efficient alternative.

Consider a state-dependent, pay-go policy that pays $T^* - \epsilon$ to the old when they experience an $H$-shock and $T^* + \epsilon$ when experiencing an $L$-shock, where $1 \gg \epsilon > 0$. With a constant pay-go policy paying $T^*$ already in place, announcing a change to this state-dependent policy one period ahead does not affect the current old and improves the ex ante welfare of younger generations. Expected utility of unborn generations now depends on the aggregate state when young and when old:

$$EU_t = \frac{1}{4} C_{t+1,HH}^{1-\gamma} + \frac{1}{4} C_{t+1,HL}^{1-\gamma} + \frac{1}{4} C_{t+1,LL}^{1-\gamma} + \frac{1}{4} C_{t+1,LL}^{1-\gamma},$$

(12)

where,

$$C_{t+1,HH} = (1 - (T^* - \epsilon)) RH + (T^* - \epsilon),$$
$$C_{t+1,HL} = (1 - (T^* - \epsilon)) RL + (T^* + \epsilon),$$
$$C_{t+1,LL} = (1 - (T^* + \epsilon)) RH + (T^* - \epsilon),$$
$$C_{t+1,LL} = (1 - (T^* + \epsilon)) RL + (T^* + \epsilon).$$

(13)

The marginal impact of increasing $\epsilon$ from zero is given by

$$\frac{\partial EU_t}{\partial \epsilon} \bigg|_{\epsilon=0} = \frac{1}{2} ((1 - T^*) RL + T^*)^{-\gamma} - \frac{1}{2} ((1 - T^*) RH + T^*)^{-\gamma} > 0,$$

(14)

which holds for the unborn as well as the current young (independent of the current state). This shows that the state-dependent, bilateral intergenerational redistribution ex ante Pareto dominates the constant pay-go scheme.4

6 Pay-Go Policy Uncertainty

Here we maintain the assumption of aggregate return risk, assume a pay-go policy is already in place, and add policy risk, namely the possibility that the pay-go policy will be immediately

4Note that from an ex interim perspective the state-dependent pay-go does not Pareto improve relative to the constant pay-go policy. The ex- interim criterion considers agents born at $i$ in $H$ versus $L$ as separate entities that both have to be made better off for a Pareto improvement. Yet, increasing $\epsilon$ cannot increase the utility of an agent born in $H$ if a transfer of $T^*$ is in place and the interest rate is, thus, $S = 1$. If, however, $T < T^*$ and accordingly $S < 1$, raising $\epsilon$ might very well even ex-interim Pareto improve, depending on parameters.
terminated. In particular, each period consumption can take one of the following four equally likely values, corresponding to high returns and pay-go retained (HR), low return and pay-go retained (LR), high returns and pay-go terminated (HT), and finally low returns and pay-go terminated (LT):

\[
\begin{align*}
C_{t+1,HR} &= \alpha (1 - T)S + (1 - \alpha)(1 - T)RH + T, \\
C_{t+1,LR} &= \alpha (1 - T)S + (1 - \alpha)(1 - T)RL + T, \\
C_{t+1,HT} &= \alpha (1 - T)S + (1 - \alpha)(1 - T)RH, \\
C_{t+1,LT} &= \alpha (1 - T)S + (1 - \alpha)(1 - T)RL.
\end{align*}
\]

The state-dependent consumption levels, given in (15), now incorporate states in which pay-go is randomly eliminated to the cost of the contemporaneous elderly. This places government-generated policy uncertainty in high relief. We again use the agents’ first order condition at \( \alpha = 0 \) and obtain

\[ S = R \frac{HA + LB}{A + B}, \]  

where

\[
\begin{align*}
A &= [(1 - T)RH + T]^{-\gamma} + [(1 - T)RH]^{-\gamma} \\
B &= [(1 - T)RL + T]^{-\gamma} + [(1 - T)RL]^{-\gamma}.
\end{align*}
\]

For \( T = 0 \), the safe rate \( S \) is the same as in the basic model and given by (4). As \( T \) goes to 1, \( S \) converges to the very same value:

\[ \lim_{T \to 1} S = R \frac{HL^\gamma + H^\gamma L}{H^\gamma + L^\gamma}. \]

However, the safe rate does not stay constant – it rises as \( T \) is increased from zero:

\[ \frac{\partial S}{\partial T} \bigg|_{T=0} = \frac{\gamma (HL)^{\gamma-1}(H - L)^2}{(H^\gamma + L^\gamma)^2} > 0. \]

Thus, as \( T \) goes from zero to one, \( S \) first rises and then falls. When utility is logarithmic, \( S = RHL \) at the extremes. For our standard parameters with \( RHL = 0.57 \) the safe rate \( S \) peaks at 0.70 for \( T = 0.65 \). Numerically, values of \( T \) above 0.65 make pay-go look cheaper with \( S \) dropping to 0.57. Hence, above 0.65, increasing \( T \) – expanding pay-go – makes pay-go appear cheaper as it becomes economically more damaging. For higher risk aversion, \( S \) is lower and peaks earlier, but the pattern with respect to increases in \( T \) stays the same. Take \( \gamma = 2 \), then \( S = 0.31 \) at the extremes and peaks for \( T = 0.44 \).

Eliminating or dramatically reducing pay-go may reflect a decision to raise output or preclude an unsustainable policy of ever increasing pay-go. A gradual reduction in the scale of pay-go starting at an uncertain time would, of course, have different implications for the path of \( S \) compared to the immediate reduction we contemplate.

The reason for this surprising result is that as \( T \) approaches 1 agents only care about the low-consumption states HT and LT. Consumption in these states is, however, just a scaled down (by factor \( 1 - T \)) version of the two possible states when \( T = 0 \). Given the homothetic preferences, this implies the same risk-free rate.
How does a higher value of \( T \) impact welfare? Clearly, welfare always declines as \( T \) approaches one. However, when increasing \( T \) from zero, we get

\[
\left. \frac{\partial EU_t}{\partial T} \right|_{T=0} = \frac{1}{4} \left( (1 - 2RH)(RH)^{-\gamma} + (1 - 2RL)(RL)^{-\gamma} \right)
\]

\[
= \frac{1}{4} \left( 1 - 2R \frac{H(RH)^{-\gamma} + L(RL)^{-\gamma}}{(RH)^{-\gamma} + (RL)^{-\gamma}} \right) \left( (RH)^{-\gamma} + (RL)^{-\gamma} \right)
\]

\[
= \frac{1}{4} \left( 1 - 2S \right) \left( (RH)^{-\gamma} + (RL)^{-\gamma} \right),
\]

which is, in contrast to the case without policy risk, negative as long as \( S > 0.5 \) at \( T = 0 \) (which is true, for instance, in our numerical example). Intuitively, an otherwise Pareto-improving paygo policy can make all generations worse off if it is subject to the risk of abrupt termination, as long as the economy is not in too much desperate need for (non-policy) risk mitigation.

7 Private Borrowing Rates Exceeding Lending Rates

Our last model variant restores macro-return risk with no potential for policy reversal. Instead, this model features two safe rates – a low lending rate and a high borrowing rate, with the wedge between the two being driven by transaction costs. Each generation contains \( A \)- and \( B \)-type workers in equal proportion. \( A \) workers earn 1 when young and 0 when old. \( B \) workers earn 0 when young and 1 when old. \( A \) workers consume when old. \( B \) workers consume when young. Let \( \delta > 1 \) denote a transaction cost wedge between the borrowing rate, \( S\delta \), and the lending rate, \( S \). Both types have logarithmic utility. In equilibrium, the total lending of the \( A \)s, \( \alpha_A \), equals the total borrowing of the \( B \)s, \( 1/(S\delta) \), who borrow in full against their future wages. The \( A \)s maximize

\[
EU_{A,t} = \frac{1}{2} \log [\alpha_A S + (1 - \alpha_A)RH] + \frac{1}{2} \log [\alpha_A S + (1 - \alpha_A)RL].
\]

Optimal \( \alpha_A \) satisfies

\[
2\alpha_A = \frac{RH}{RH - S} + \frac{RL}{RL - S}.
\]

In equilibrium, the risk-free rate \( S \) is therefore determined by

\[
\frac{RH}{RH - S} + \frac{RL}{RL - S} = \frac{2}{S\delta}.
\]

For illustration, re-consider the case of \( R = 3, L = 0.1, \) and \( H = 1.9 \). Without the \( B \) type we would have a yearly risk-free rate of \(-1.9\%\). Take \( \delta = 3 \), which corresponds to a yearly wedge of about \( 3.7\% \). In the equilibrium with both types the yearly lending rate is \(-0.2\%\) with a corresponding borrowing rate of \( 3.5\% \). If there were no wedge, i.e., were \( \delta = 1 \), the safe rate would be \( 1.7\% \), showing that aggregate risk combined with financial frictions drive the safe rate below the growth rate in this example.

Now suppose that parameters are such that \( S < 1 < \delta S \) in equilibrium. Moreover, the government can borrow at the lending rate and, observing that it can borrow at negative rates,
adopts pay-go policy at scale $T$. It can easily be seen that the total amount of lending by $A$s is given by $(1 - T)\alpha(T)$, where from the first order conditions we obtain

$$2(1 - T)\alpha(T) = \frac{(1 - T)RH + T}{RH - S} + \frac{(1 - T)RL + T}{RL - S}.$$  \tag{23}

Given that $RL < 1 < RH$, the supply curve for lending shifts downwards in $T$. Since the borrowing of the $B$ type increases with $T$, the pay-go policy always increases the risk-free rate. This necessarily makes the lender – the $A$ type – better off. However, it makes the $B$s worse off: In addition to having to borrow at a rate that is greater than 1 to cover the size $T$ units of tax, they also have to borrow at higher rates. Hence, the observation that one group’s safe rate is below the growth rate provides no basis, even with linear technology, for suggesting pay-go policy is free. Comparing “the” safe rate with the growth rate is fallacious since there are two safe rates – one above and one below the growth rate. What about running pay-go by having the government take just from the $A$s and give just to the $A$s? This helps current and future $A$s, but it hurts the $B$s by reducing the supply of loans, raising equilibrium $S$ and, thus, the borrowing rate, $\delta S$.

8 Conclusion

Safe rates that average less than growth rates make deficit finance alluring. But low safe rates can reflect, among other things, incomplete intragenerational risk-sharing, incomplete intergenerational risk-sharing, government-generated uncertainty, or credit market imperfections. In all such cases, pay-go policy, a.k.a. deficit finance, is not free. It redistributes across generations or within generations. And if pay-go does Pareto improve, it may reflect second-best policy that leaves young and future generations worse off relative to enacting first best policy – policy that addresses the root cause of low safe rates. Moreover, uncertainty about the resolution of government debt policies can, itself, lower the government’s borrowing rate, making deficits look cheaper precisely when they are becoming economically more expensive.

References


\footnote{If the government could, however, compensate the young $B$s for the increased interest rate through transfers from the young $A$s (e.g. via subsidized borrowing) a Pareto improvement would be possible. This can easily be shown by an argument similar to equation (8) where the transfer is chosen to keep constant the net financial flows between $A$s and $B$s.}


Reis, R. (2021). The constraint on public debt when r<g but g<m. Working Paper, London School of Economics.


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