Re-use of Collateral: Leverage, Volatility, and Welfare*

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Abstract

We assess the quantitative implications of collateral re-use on leverage, volatility, and welfare within an infinite-horizon asset-pricing model with heterogeneous agents. In our model, the ability of agents to reuse frees up collateral that can be used to back more transactions. Re-use thus contributes to the buildup of leverage and significantly increases volatility in financial markets. When introducing limits on re-use, we find that volatility is strictly decreasing as these limits become tighter, yet the impact on welfare is non-monotone. In the model, allowing for some re-use can improve welfare as it enables agents to share risk more effectively. Allowing re-use beyond intermediate levels, however, can lead to excessive leverage and lower welfare. So the analysis in this paper provides a rationale for limiting, yet not banning, re-use in financial markets.

Keywords: Heterogeneous agents, leverage, re-use of collateral, volatility, welfare.

JEL Classification Codes: D53, G01, G12, G18.

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1 Introduction

Re-use and re-hypothecation of collateral has become a major activity in financial markets. It refers to the practice of financial institutions to reuse collateral received in one transaction for another transaction.[1] Global collateral re-use at the onset of the financial crisis was estimated to be as much as US $6.6 trillion, and as dropping to US $3.4 trillion at the end of 2010; see Singh (2011).[2] While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. For example, the Vice-President of the ECB, Vítor Constâncio, has stressed that "activities of re-hypothecation and re-use of securities amplified the creation of chains of inside liquidity and higher leverage".[3] Moreover, the Financial Stability Board (FSB) recently published work that analyzes the financial stability implications of collateral re-use; see FSB (2017a). Several regulatory frameworks already foresee specific rules such as limits and transparency requirements for re-use of (non-cash) collateral. For example, European retail investment funds (UCITs) are banned from reusing non-cash collateral. Moreover, the EU framework for margin requirements for non-centrally cleared derivatives foresees a ban on re-use of non-cash collateral that is posted as an initial margin. Given the importance of collateral re-use in financial markets and the need to inform ongoing regulatory initiatives, it is important to develop a model framework to understand the (quantitative) implications of collateral re-use on financial market outcomes and welfare.

In this paper, we present an asset-pricing framework with heterogeneous agents in which financial securities are only traded if the promised payments associated with selling these securities are backed by collateral. To generate collateralized borrowing in equilibrium, we assume that there are two types of agents who differ in risk aversion and in their beliefs about the likelihood of bad shocks to the economy. The agent with the low risk aversion and optimistic beliefs (agent 1) is the natural buyer of risky assets and takes up leverage to finance these investments. The agent with the high risk aversion and pessimistic beliefs (agent 2) has a strong desire to insure against bad shocks and is thus willing to buy bonds, thereby providing financing to the other agent. We introduce re-use of collateral in this setting by allowing agents who receive securities as collateral to sell these securities to other agents. As a consequence, the security can again be used to collateralize transactions, allowing agents to further build up their leveraged position in the risky security.

We first illustrate this leverage effect of collateral re-use qualitatively in a simple two-period version of our model. In this setting, we also analyze the effects of restricting re-use on collateralized borrowing, leverage, and welfare. Subsequently, we present an infinite-horizon model in which agents

[1] Institutions typically receive collateral in securities financing transactions (SFTs; e.g., reverse repo, securities lending/borrowing) or derivative transactions, and if eligible for re-use, may post it as collateral in other transactions (e.g., for repos, securities lending/borrowing, derivatives collateral) or use if for short sales. Re-hypothecation typically refers to the use of client assets to obtain funding to finance client activities and is considered a subset of re-use activity.

[2] In a related and more recent study, Kirk et al. (2014) document that three large US dealer banks have a strong dependency on re-use of collateral for financing their activities, estimating their total amount of collateral re-use to be approx. US $1.3 trillion at end-2013.

[3] V. Constâncio, "Beyond traditional banking: a new credit system coming out of the shadows", Speech, University of Frankfurt, 17 October 2014.

have Epstein–Zin utility with identical time discount factors, identical inter-temporal elasticity of substitution (IES) parameters, yet differing risk aversion parameters, agent 1 being less risk averse than agent 2. Moreover, agents (agree to) disagree about the likelihood of disaster shocks. We calibrate these low-probability events based on Barro and Ursúa (2008) and assume that agents' beliefs about their likelihood deviate from the objective probabilities in opposite directions, agent 1 being optimistic and agent 2 being pessimistic.

In the dynamic model, the amount of collateral needed to back a transaction is determined in equilibrium. When the economy is hit by a bad shock, the leveraged agent, agent 1, loses financial wealth. As a result, the collateral constraint forces him to reduce consumption and to sell risky assets to the risk-averse agent. These actions trigger an additional decrease in asset prices, which further reduces the wealth of agent 1—thereby reinforcing the impact of the bad shock on the wealth distribution and on asset prices.

Allowing for re-use of collateral in the economy increases the available collateral in financial markets, thus agents can build up leverage far beyond what is permissible in economies in which re-use is prohibited. These highly leveraged endogenous asset portfolios lead to large movements in the wealth distribution when good or bad shocks hit. For this reason, our calibrated model with unlimited re-use can generate first and second moments of risk-free and risky returns as in US financial market data even for values of the IES below 1 and for risk-aversion parameters far less than 10.

In the next step of our analysis we introduce re-use limits to constrain the buildup of leverage in agents' asset portfolios. Compared to the benchmark economy with free re-use, these limits restrict agents' portfolio holdings in equilibrium. In particular, the optimistic and less risk-averse agent in the economy, agent 1, cannot build up as much leverage as in the benchmark economy. Therefore, the impact of negative shocks on this agent's wealth share is greatly reduced. As a consequence, the volatility of the wealth distribution shrinks considerably and so does the volatility of financial asset returns. This reduction in volatility is much stronger than the accompanying decrease in the equity premium. As a result, both the realized average Sharpe ratio in the economy as well as agents' anticipated Sharpe ratios under their subjective beliefs increase strongly in response to limiting re-use.

We complete the analysis of the effects of re-use limits on financial-markets equilibria with an examination of welfare implications. In our welfare analysis, we consider unanticipated changes in regulation and find that intermediate levels of re-use limits are welfare optimal. Compared to very loose or very strict regulation, the welfare of one agent is increased when transfers are chosen such that the other agent's welfare is kept constant; in some cases we observe Pareto improvements even without transfers. The relation between re-use limits and welfare is non-monotone because two counteracting forces are at play: First, the ability to reuse allows for more risk sharing in the economy. This is generally beneficial for welfare given the agents' heterogeneity in risk aversion. Second, the heterogeneity in agents' beliefs triggers agents to build up leveraged positions beyond what is needed to optimally share risks. As the ability to reuse collateral allows agents to build up this leverage, limiting re-use has the potential to steer agents' choices toward socially optimal levels.

In our model with two assets and two types of agents, the only collateral that can be reused is equity and the only way in which it is reused in equilibrium is for short sales. In reality there are many other ways to reuse collateral, yet some of these have effects similar to short sales when it comes to increasing exposure to risk, overall leverage, and volatility of returns. For instance, the collateral asset might be reused as margin in derivative trades, which may generate a negative exposure to the market, as short selling does. More generally, re-use of collateral leads to an increase in the availability of collateral in the financial system, ultimately allowing entities to enter more transactions and more leveraged positions than without re-use (see also Section 2). We therefore view our model as a "reduced form" of a more detailed model with a variety of assets and a variety of possibilities for re-use.

There is a growing literature on the role of re-use and re-hypothecation in financial markets. Singh and Aitken (2010), Singh (2011), and Kirk et al. (2014) use publicly available data to estimate the amount of collateral re-use in financial markets. Bottazzi et al. (2012), Andolfatto et al. (2017), Maurin (2015), and Gottardi et al. (2015) present theoretical models of collateral re-use. In particular, Bottazzi et al. (2012) provide a general theory of re-use in repo markets whereas Andolfatto et al. (2017) show how re-use may benefit the provision of liquidity in financial markets. Maurin (2015) focuses on the role of re-use in completing markets, whereas Gottardi et al. (2015) discuss how re-use may impact collateral constraints and haircuts. Furthermore, Infante (2018), Infante and Vardoulakis (2018), and Eren (2014) present models that consider the funding role of re-use for dealer banks. Eren (2014) shows how re-use may expose a hedge fund to a dealer's default, whereas Infante (2018) and Infante and Vardoulakis (2018) consider how collateral runs may arise due to reuse. However, none of these papers provide a quantitative analysis of the implications of re-use for aggregate financial market outcomes. Furthermore, with the exception of Andolfatto et al. (2017), who discuss limits on re-use in a monetary model and find that limits need to be stricter in economies with lower inflation, none of these papers focuses on the implications of regulating re-use.

The economic literature on the effects of collateralized borrowing and asset market volatility has been very active and continues to grow; see, among many other papers, Geanakoplos (1997), Aiyagari and Gertler (1999), Coen-Pirani (2005), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009), Garleanu and Pedersen (2011), and Fostel and Geanakoplos (2013). Rytchkov (2014), Chabakauri (2013), and Brumm et al. (2015a) analyze the volatility implications of collateral constraints in general equilibrium models with heterogeneous agents and two assets, while Brumm et al. (2015b) investigate the welfare implications of margin regulation in such a model.

The remainder of this paper is organized as follows. Section 2 provides an overview of industry practices for collateral re-use. Section 3 presents a simple two-period model in which we can observe some of the qualitative features of re-use and its regulation. In Section 4 we describe our infinite-horizon model and its benchmark calibration. Section 5 presents numerical results for the impact of re-use on leverage and volatility in the benchmark economy and a discussion of the economic mechanism. In Section 6 we examine the welfare implications of re-use limits. Section 7 concludes. The Appendix contains additional results.

2 An Overview of Collateral Re-use Practices

Since the academic literature on the re-use of collateral is still young, we begin with a brief description of re-use in industry practice. For this purpose, we first define the terms re-use and re-hypothecation. Next we provide some figures on the size of global re-use activities. Then we discuss some regulatory concerns about the effects of collateral re-use on the stability of financial markets. We complete our overview with a look at recent initiatives at the international level assessing financial stability risks and benefits associated with collateral re-use.

2.1 Defining Re-use of Collateral

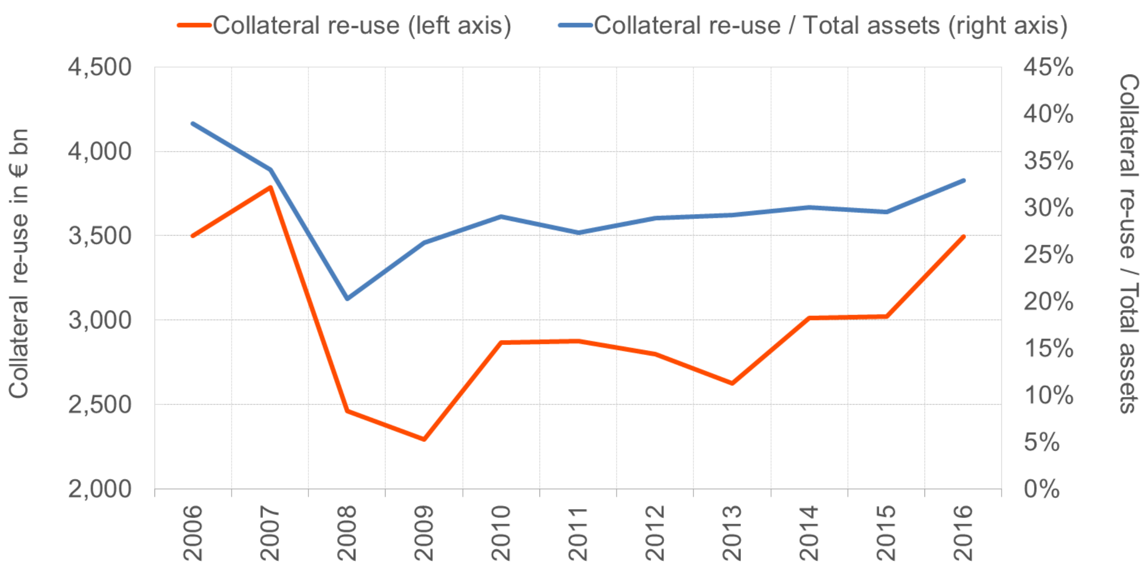
The Financial Stability Board (FSB) defines collateral re-use in a broad sense as "any use of assets delivered as collateral in a transaction by an intermediary or other collateral taker"FSB (2017b). Financial institutions receive collateral in securities financing transactions (SFTs; e.g., reverse repo, securities lending/borrowing) or derivative transactions. Depending on the terms of the transaction, the collateral may be eligible for re-use by the receiving counterparty: they can use it for their own purposes (e.g., for repos, securities lending/borrowing, short sales, derivatives collateral). For example, in a repurchase transaction (repo), the counterparty providing cash (i.e., the collateral taker) can reuse the securities obtained as collateral to, among other things, pledge as collateral in a separate transaction with a third party. Often, the term "collateral re-use" is used interchangeably with the term "re-hypothecation". Again, we follow the FSB (2017b), where re-hypothecation is defined narrowly as "any use by a financial intermediary of client assets". We therefore consider re-hypothecation as a special case of the broad collateral re-use concept. In a typical re-hypothecation transaction, securities that serve as collateral for a secured borrowing (e.g., a margin loan extended to a hedge fund) are further used by the intermediary to obtain funding for the initial transaction.

2.2 The Relevance of Collateral Re-use in Financial Markets

Re-use of collateral has become a major activity in financial markets and is a common practice across many entities in the financial system. Though incomplete data makes it difficult at this juncture to pin down the exact size of global collateral re-use activity, publicly available data on the collateral re-use activity of globally active banks indicates that collateral re-use plays an important role in financial markets. Figure I displays the evolution of re-use activity for a set of 11 global banks. The level of collateral re-use among these 11 global banks amounted to around 3.8 trillion euros and approximately 30 percent of the total assets of these financial institutions before the crisis.[4] The time series further reveals that collateral re-use exhibits strong pro-cyclicality. The most striking fact appearing in the data is the sharp contraction in collateral re-use taking place during the global financial crisis, in 2008. Thereafter, the amount of collateral reused by financial institutions has risen again, without, however, returning to pre-crisis levels.

[4]See Singh (2014) for a similar exercise using a different methodology.

Figure I: Collateral re-use



Own calculations based on 11 banks' annual reports. The institutions are Deutsche Bank, BNP Paribas, Credit Suisse, UBS, Barclays, RBS, HSBC, Nomura, Morgan Stanley, JPMorgan Chase & Co., and Goldman Sachs. The figure shows collateral re-use in billions of euros and the ratio of the values of reused collateral and total assets.

2.3 Financial Stability Risks Associated with Collateral Re-use

While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. For example, the same piece of collateral may be used to back a chain of transactions and may therefore contribute to system-wide leverage. This concern has, for example, been raised by Vice-President of the ECB Vítor Constâncio, who stressed that "activities of re-hypothecation and re-use of securities amplified the creation of chains of inside liquidity and higher leverage."[5] More specifically, the use of the same security as underlying collateral for different transactions increases the sum of exposures in the financial system and, and as result, creates leverage across the intermediation chain. The evidence presented in Figure I suggests that re-use practices may be significant drivers of financial system leverage. The apparent cyclical behavior of aggregate re-use activities suggests that re-use may also contribute to pro-cyclicality in the financial sector. In good times, market participants tend to be more willing to allow counterparties to reuse collateral, increasing market liquidity and lowering the cost of capital. However, in stressed market conditions, market participants become more sensitive to counterparty risk and hence refuse to allow their collateral to be reused, amplifying strains already present in markets (see also FSB (2017b)). The sharp contraction of aggregate collateral re-use levels shown in Figure I indeed suggests that pro-cyclicality could be an important risk channel. Moreover, the FSB (2017b) highlights the issue

[5] V. Constâncio, "Beyond traditional banking: a new credit system coming out of the shadows", Speech, University of Frankfurt, 17 October 2014.

of interconnectedness arising from chains of transactions involving the re-use of collateral. Large exposures among financial institutions create a risk of contagion. The unwinding of a transaction by one institution may trigger the unwinding of transactions by other institutions, and lead to the propagation of shocks through the financial system. Finally, in the context of re-hypothecation it has been argued excessive reliance on this practice for funding purposes may create the possibility of a the run on a prime broker if there are concerns about its credit worthiness, and therefore clients have an incentive to withdraw their assets from their prime brokers.[6] This risk appears, at least to some extent, to have materialized when Bear Stearns and Lehman Brothers were running into trouble and hedge funds moved their assets from their prime brokers. These actions exacerbated the financing problems faced by the two entities, which had relied on the availability of client assets for the financing of their activities.[7]

2.4 The Regulatory Framework and Ongoing Initiatives

When it comes to the regulatory framework for collateral re-use, it helps to distinguish between broader re-use of collateral and the narrower set of client asset re-hypothecation. On collateral re-use more broadly, no general restrictions exist. There are, however, a few cases where limits or bans on re-use exist for specific entities or transactions. One example relates to retail investment funds (UCITS funds) in the EU. The ESMA Guidelines on ETFs and other UCITS issues[8] foresee that UCITS do not reuse non-cash collateral they receive and set conditions for the reinvestment of cash collateral. A second example relates to the initial margin posted in OTC derivatives. The EU regulatory technical standards (RTS) on margin requirements for non-centrally cleared derivatives foresees that re-use of non-cash collateral collected as an initial margin shall not be permitted. In the international context, the FSB has recently published two papers related to collateral re-use. One (FSB (2017b)) analyzes the financial stability implications of collateral re-use as well as the benefits of collateral re-use for financial markets. This document furthermore describes a number of post-crisis regulatory reforms related both directly and indirectly to the re-use of collateral and mitigating at least some of the risks described above. In particular, rules affecting collateral transactions such as repos may indirectly limit the extent to which entities may reuse collateral. For example, the FSB (2017b) highlights that the Basel III leverage ratio (LR) framework indirectly incentivizes banks to keep re-use activity below excessive levels. The LR framework triggers regulatory capital for any additional funding raised by reusing collateral, and therefore is considered as an important brake with which to address risk related to the contribution of re-use of collateral to the buildup of leverage in the banking sectorFSB2017a. While the FSB sees no need for immediate additional regulatory action, it considers that appropriately monitoring collateral re-use at the global level will be an important step toward obtaining a clearer understanding of global collateral re-use activities. Hence,

[6] See Duffie (2010) for an extensive discussion of this issue and Infante and Vardoulakis (2018) for a theoretical model exploring this risk channel.

[7] See Aragon and Strahan (2012) for an analysis of the impact the failure of Lehman Brothers had on its hedge fund clients.

[8] See `https://www.esma.europa.eu/sites/default/files/library/2015/11/esma-2014-0011-01-00_en_0.pdf`; last accessed October 19, 2018.

to enhance monitoring of global collateral re-use activity, the FSB published a paper that sets out a methodology for measuring non-cash collateral re-use, and describes the related data elements that national authorities would report to the FSB (see FSB (2017a)). This FSB paper further defines various metrics that could be used to monitor financial stability risks associated with collateral re-use.

To address financial stability concerns related to re-hypothecation of client assets, the FSB issued several recommendations in 2013 (see FSB (2013)). To address the aforementioned potential excessive reliance on re-hypothecation for funding purposes, the recommendations aim at reducing client uncertainty about the extent to which assets have been re-hypothecated and treatment in the case of bankruptcy,[9] and at limiting re-hypothecation of client assets to financial intermediaries subject to adequate regulation of liquidity risk. Moreover, the FSB recommended that prime brokers/banks shall use only that amount of client assets needed for the purposes of financing client activities, representing an aggregate limit on the amount of client assets they can re-hypothecate. Without this aggregate limit, prime brokers could in theory use all client assets eligible for re-hypothecation, including for their own activities and irrespective of the aggregate level of financing their set of clients requires. The combination of these measures are expected to reduce the likelihood of clients suddenly removing assets their prime brokers have relied on for funding their own business, as it was the case with Bear Stearns and Lehman Brothers.

With regard to explicit limits on re-hypothecation already existing in regulatory frameworks, in the US an individual limit of 140 percent of the client's indebtedness applies, next to an aggregate limit in line with the aforementioned FSB recommendation. In the EU neither an aggregate nor a client-individual limit on the amount of client assets available for re-hypothecation exists and is instead agreed between the two counterparties. However, specifically to EU investment funds, custodians of UCITs assets are prohibited from re-using (including, but not limited to, transferring, pledging, selling, and lending) funds' assets for their own account (see FSB (2017a)).

3 A Simple Two-Period Model

In this section we present a simple two-period model to illustrate some of the key qualitative effects of collateral re-use. Our objective is to develop some initial intuition for our results in the calibrated infinite-horizon economy.

3.1 The Physical Economy

Consider a two-period model with two agents, $h = 1, 2$, and a single consumption good. In period 0, the economy is in state $s = 0$. There are two possible states in period 1, $s = 1, 2$. The aggregate endowment of the good, ω_s, is state-dependent and has the property $\omega_2 > \omega_0 > \omega_1$. So, we can call state 2 "good" and state 1 "bad". In state 0, the individual endowments of the two agents are

[9] For their implementation in the EU, see the recently agreed Securities Financing Transaction Regulation (SFTR) imposes minimum market-wide conditions to be met on re-hypothecation of client assets, such as prior consent, disclosure of the risks and consequences of re-hypothecation, and transfer of the financial instruments from the account of the client.

comprised of two parts: first, identical individual incomes, $\frac{1}{2}(1-\delta)\omega_0$; and second, identical individual portions of the dividends from a "Lucas tree", $\frac{1}{2}\delta\omega_0$, as a result of each agent initially holding half of the tree. So, the tree is in unit net supply and dividends constitute the portion $\delta \in (0,1)$ of the aggregate endowment, $d_s = \delta\omega_s$. In states $s = 1, 2$, the endowments of the two agents are given by their identical individual incomes, $\frac{1}{2}(1-\delta)\omega_s$. Whether an agent receives any dividend income depends on that agent's asset portfolio. In the absence of any trades in period 0, each agent would also receive the dividend income $\frac{1}{2}\delta\omega_s$ in state $s = 1, 2$.

In period 0, the agents can trade a risk-free bond (asset 1) in zero net supply—in addition to the tree (asset 2). We denote the period-0 asset prices by p and q for the bond and the tree, respectively, and the agents' post-trading asset holdings by $\left(\phi^h, \theta^h\right)$. Agent h's budget constraints are

$$\begin{aligned} c_0^h &= \frac{1}{2}\omega_0 - p\phi^h - q\left(\theta^h - \frac{1}{2}\right), \\ c_s^h &= \frac{1}{2}(1-\delta)\omega_s + \phi^h + \theta^h\delta\omega_s, \quad s = 1, 2. \end{aligned}$$

To present the main qualitative features of risk-sharing patterns, portfolio choices, and welfare in the simplest possible framework, we assume that agent 1 is risk neutral. His utility function is

$$U^1(c) = c_0 + \pi^1 c_1 + (1-\pi^1)c_2,$$

with $\pi^1 \in (0,1)$ denoting his subjective probability for state 1. Contrary to agent 1, agent 2 is risk averse. She has log utility,

$$U^2(c) = \ln c_0 + \pi^2 \ln c_1 + (1-\pi^2)\ln c_2,$$

with a subjective probability $\pi^2 \in (0,1)$ for state 1.

3.2 Arrow–Debreu Equilibrium

Without any additional constraints, markets are complete and there is a unique Arrow–Debreu equilibrium. The first-order conditions of agent 1's utility maximization problem determine the state prices for the three states, $s = 0, 1, 2$, in the economy,[10] which are, respectively, $1, \pi^1$, and $1 - \pi^1$. Solving agent 2's utility maximization problem at these state prices yields her consumption allocation

$$\begin{aligned} c_0^2 &= \frac{1}{4}\left(\omega_0 + \pi^1\omega_1 + (1-\pi^1)\omega_2\right), \\ c_1^2 &= \frac{\pi^2}{4\pi^1}\left(\omega_0 + \pi^1\omega_1 + (1-\pi^1)\omega_2\right), \\ c_2^2 &= \frac{1-\pi^2}{4(1-\pi^1)}\left(\omega_0 + \pi^1\omega_1 + (1-\pi^1)\omega_2\right). \end{aligned}$$

[10] We consider only parameter settings that result in an Arrow–Debreu equilibrium in which agent 1's consumption is positive in every state. This property can always be guaranteed, for instance, if the difference between the subjective probabilities π^1 and π^2 is sufficiently small (when all other parameters are given).

Agent 1's first-order conditions also determine the two asset prices in period 0,

$$p = 1, \ q = \pi^1 \delta\omega_1 + (1 - \pi^1)\delta\omega_2.$$

We assume that both states in period 1 are equally likely. The risk-neutral agent agent 1, has correct beliefs, so $\pi^1 = \frac{1}{2}$. The risk-averse agent, agent 2, is pessimistic and believes that the bad state is more likely than the good state, so $\pi^2 > \frac{1}{2}$. Observe that this assumption implies that $c_1^2 > c_0^2 > c_2^2$ for agent 2's consumption allocation.

To simplify the subsequent analysis of collateral re-use, we consider a specific parameterization of the two-period economy with

$$\omega_s = (1, 0.9, 1.1), \ \delta = 0.4, \ \pi^2 = \frac{21}{40}.$$

The resulting consumption allocations in the Arrow–Debreu equilibrium are

$$c^1 = \left(\frac{1}{2}, \frac{15}{40}, \frac{25}{40}\right) \quad \text{and} \quad c^2 = \left(\frac{1}{2}, \frac{21}{40}, \frac{19}{40}\right).$$

Equilibrium asset prices and portfolios are

$$(p, q) = \left(1, \frac{2}{5}\right) \quad \text{and} \quad (\phi^1, \theta^1) = \left(-\frac{3}{4}, \frac{19}{8}\right), \quad (\phi^2, \theta^2) = \left(\frac{3}{4}, -\frac{11}{8}\right),$$

respectively.

3.3 Collateral Constraints

The Arrow–Debreu equilibrium assumes that agents will honor any debt in the second period. We now remove this assumption and instead impose collateral constraints on the two agents. The agents can hold only portfolios that do not give them an incentive to default in period 1—that is, the net portfolio payoff of each agent must be positive in both states:

$$\phi^h + \theta^h \delta\omega_s \geq 0, \quad s = 1, 2.$$

If the portfolios supporting the Arrow–Debreu equilibrium allocation satisfy these constraints, then the equilibrium prices and portfolios also constitute the (financial-markets) equilibrium of the collateral-constrained economy.

In our parameterization, the risk-neutral agent 1 is long in the tree and so, if any collateral constraint may bind, then it would be the constraint in the bad state, $s = 1$. The risk-averse agent, agent 2, is short in the tree and so the constraint in the good state, $s = 2$, is tighter than that in the bad state. For the specific values,

$$\phi^1 + \theta^1 \delta\omega_1 = \frac{21}{200} \quad \text{and} \quad \phi^2 + \theta^2 \delta\omega_2 = \frac{29}{200}.$$

Therefore, the Arrow–Debreu equilibrium is also the equilibrium of the economy with collateral constraints. Simply put, the collateral constraints are sufficiently "loose" and so do not prevent the complete-markets equilibrium.

3.4 The Re-use Constraint

Finally, we impose a crucial additional restriction on the agents in the economy. Short positions in the tree (asset 2) cannot be "naked"— that is, an agent must borrow the tree in order to enter a short sale. In particular, an agent can only enter a short sale of the tree if the other agent provided him or her with tree shares as re-usable collateral. However, the agent cannot reuse all of the received collateral but only the portion $\kappa \in [0,1]$. This condition constitutes a collateral re-use constraint.

In equilibrium, the risk-averse agent, agent 2, always enters into a short position in the tree (asset 2) and holds a long position in the bond (asset 1); see, by way of illustration, the portfolios supporting the Arrow–Debreu equilibrium in the numerical example in Section 3.2. Therefore, it suffices to analyze a re-use constraint for agent 2 only.

When agent 1 borrows (in the bond) from agent 2, then he must pledge $\frac{-\phi^1}{\delta\omega_1} > 0$ shares of the tree as collateral to agent 2. And now agent 2 faces the re-use constraint

$$\theta^2 \geq \kappa \frac{\phi^1}{\delta\omega_1}$$

when she wants to reuse the collateral to enter a short sale in the tree. Observe that the right-hand side of the constraint is negative. It constitutes a lower bound on the possible short sales for agent 2. If the re-use parameter is zero, $\kappa = 0$, then not only is re-use prohibited but so are short sales of the tree.

For our illustrative parametrization, the asset holdings supporting the Arrow–Debreu allocation also constitute the portfolios of the (financial-markets) equilibrium for the economy with collateral and re-use constraints if and only if they satisfy the re-use constraint

$$-\frac{11}{8} + \kappa\frac{25}{12} \geq 0 \quad \Longleftrightarrow \quad \kappa \geq \frac{33}{50}.$$

And so, for $\kappa \geq 0.66$, the equilibrium consumption is given by the Arrow–Debreu allocation. On the contrary, for a value of the re-use parameter below 0.66, the portfolios supporting the Arrow–Debreu equilibrium violate the re-use constraint.

3.5 Constrained Equilibrium

When the risk-averse agent 2 is lending to the risk-neutral agent 1 via a bond purchase of $\phi^2 > 0$, then agent 1 must pledge as collateral

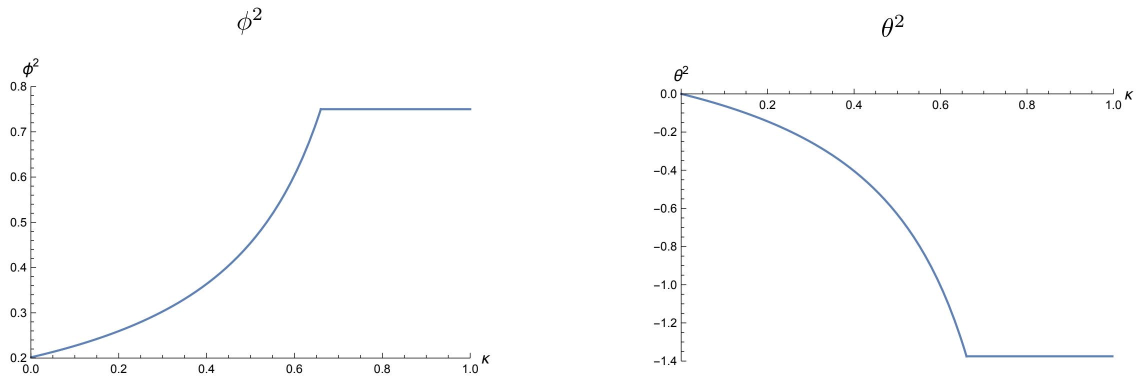
$$\frac{-\phi^1}{\delta\omega_1} = \frac{\phi^2}{\delta\omega_1}$$

shares of the tree to agent 2. If re-use is permitted in the economy, then agent 2 can use a portion κ of this collateral for short sales. If the re-use constraint is binding, then her portfolio must satisfy the condition

$$\theta^2 = \kappa\frac{-\phi^2}{\delta\omega_1}.$$

In the economy with re-use, the risk-neutral agent 1 continues to be unconstrained for all values of the re-use parameter κ. Therefore, his first-order conditions continue to determine the asset prices and they have the same values as in the Arrow–Debreu equilibrium, $(p,q) = \left(1, \frac{2}{5}\right)$. For these prices,

Figure II: Portfolio of the risk-averse agent, agent 2, as a function of κ



The figure shows graphs of the equilibrium holdings of the risk-averse agent, agent 2, as a function of the re-use parameter κ. The left plot depicts her bond position, ϕ^2, and the right graph depicts her tree holding, θ^2. For $\kappa \leq 0.66$ the re-use constraint is binding. For $\kappa > 0.66$ the re-use constraint does not bind and the holdings are identical to those supporting the Arrow–Debreu equilibrium.

we can solve agent 2's utility maximization problem to determine her optimal portfolio as a function of κ. Since the asset prices do not depend on the re-use parameter, there are no price effects when κ changes its value. As a result, the feasible regions of agent 2's utility maximization problem are a weakly increasing sequence of sets in $\kappa \in [0, 1]$.

Figure II shows graphs of agent 2's bond and tree holdings as a function of the re-use parameter κ. Note that for all values of $\kappa \in [0, 1]$, the collateral constraints are non-binding. When re-use is not permitted, $\kappa = 0$, agent 2 sells all her tree shares in period 0 and holds no shares of the tree. When κ increases, she sells as many shares of the tree short as the re-use constraint permits her to. She then invests these funds in an increasingly larger bond position. The re-use constraint is binding for $\kappa \in [0, 0.66]$. For $\kappa > 0.66$ the re-use constraint does not bind and the holdings are identical to those of the (unconstrained) Arrow–Debreu equilibrium reported above.

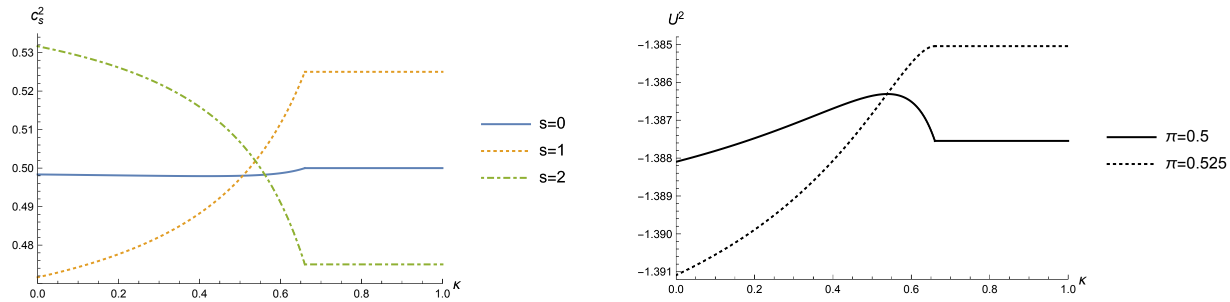
Figure III(a) shows agent 2's consumption allocations for all three states, $s = 0, 1, 2$, as a function of κ. In autarchy, agent 2 would have to consume her endowment, $(0.5, 0.45, 0.55)$. For $\kappa = 0$, the agent can trade to a slightly different allocation, $(0.4984, 0.4716, 0.5316)$. When κ increases from zero, the agent can hold increasingly larger portfolios and thereby move her consumption allocation toward the Arrow–Debreu allocation.

Figure III(b) shows agent 2's utility as a function of the re-use parameter κ—evaluated for two different beliefs. The dashed line shows the agent's utility for her subjective probability $\pi^2 = 0.525$ of state $s = 1$. Since the feasible regions of agent 2's utility maximization problems are an increasing sequence of sets for $\kappa \in [0, 0.66]$, her utility is increasing in κ in this region. Once the re-use constraint stops being binding, the utility is constant. The solid line shows her utility evaluated with the true probability $\pi^1 = 0.5$, in which case her utility is maximized for an interior value of κ.

Recall that agent 2 is pessimistic and believes that the bad state, state 1, is more likely than the good state, state 2. As a result—see Figure III(a)—she trades so that she can consume more in

Figure III: Consumption allocation and utility of the risk-averse agent, agent 2, as a function of κ

(a) c_s^2 (b) U^2



The left graph in the figure depicts agent 2's consumption levels for all three states, $s = 0, 1, 2$, as a function of κ. For $\kappa > 0.66$, the re-use constraint does not bind and the consumption allocation and utility level are identical to their respective Arrow–Debreu values. The right graph depicts the expected utility of agent 2 as a function of κ, evaluated for two different sets of probabilities. The dashed line shows the agent's utility for her subjective probability $\pi^2 = 0.525$ of state $s = 1$. The solid line shows her utility evaluated with the true probability $\pi^1 = 0.5$.

state 1 than in state 2. But when the actual probability of state $s = 1$ is smaller than her subjective probability she is moving too much consumption into that state. As a result, for $\kappa > 0.66$, the smaller the probability of the first state, the smaller her utility.

For small values of κ near zero, we observe the opposite pattern. Here the re-use constraint is so strict that the pessimistic and risk-averse agent 2 cannot build a portfolio in order to support more consumption in the bad than in the good state. And now, when the actual probability of state $s = 1$ is smaller than her subjective probability, the agent's preference for moving consumption decreases (despite her risk aversion). And so her utility increases when the probability of $s = 1$ decreases.

In sum, when the risk-averse agent 2 is overly pessimistic—that is, her subjective probability of the bad state is larger than the true probability—then she is trying to move too much consumption into the bad state. A value of the re-use parameter prohibiting the Arrow–Debreu consumption allocation now actually improves her utility (under the true probabilities). However, due to her (higher) risk aversion, agent 2 wants to smooth consumption across states in the economy more than agent 1 does, and if κ is very small, then the re-use regulation restricts the opportunity for risk sharing between the two agents too much. Thus, too little re-use restricts risk sharing in the economy; too much re-use results in an overly pessimistic agent hedging bad states too much. And so, measured under the true beliefs, we observe an interior welfare maximum for agent 2.

4 The Infinite-Horizon Model

This section introduces an infinite-horizon exchange economy with two infinitely lived heterogeneous agents trading in a Lucas tree and a bond. Agents need to post collateral to back short positions in either asset. Received collateral can be reused for collateralizing other transactions or for short-selling. A regulating agency can impose a re-use constraint to restrict the amount of received collateral that agents may reuse. Finally, the section describes the calibration strategy for the baseline model.

4.1 The Physical Economy

Time is indexed by $t = 0, 1, 2, \ldots$. Exogenous shocks (s_t) follow a Markov chain with support $\mathcal{S} = \{1, \ldots, S\}$ and transition matrix π. The evolution of time and shocks in the economy is represented by an infinite event tree Σ. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called a date-event. The symbols σ and s^t are used interchangeably. To indicate that $s^{t'}$ is a successor of s^t (or is s^t itself), write $s^{t'} \succeq s^t$. The expression s^{-1} refers to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$, there is a single perishable consumption good. The economy is populated by $H = 2$ agents, $h \in \mathcal{H} = \{1, 2\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. Agent h believes that the transition matrix of the Markov chain of exogenous shocks is π^h—this matrix may differ from the true transition matrix π. There is a single long-lived asset (Lucas tree) in the economy, which we also call stock. At the beginning of period 0, each agent h owns initial holdings $\theta^h(s^{-1}) \geq 0$ of this asset. Aggregate holdings in the long-lived asset sum to 1—that is, $\sum_{h \in \mathcal{H}} \theta^h(s^{-1}) = 1$. At date-event σ, agent h's (end-of-period) holding of the asset is denoted by $\theta^h(\sigma)$. The long-lived asset pays positive dividends $d(\sigma)$ in units of the consumption good at all date-events. The aggregate endowment in the economy is then

$$\bar{e}(\sigma) = d(\sigma) + \sum_{h \in \mathcal{H}} e^h(\sigma).$$

Agent h has preferences over consumption streams $c^h = \left(c^h(s^t)\right)_{s^t \in \Sigma}$ representable by the following recursive utility function (see Epstein and Zin (1989)):

$$U^h\left(c^h, s^t\right) = \left\{ \left[c^h(s^t)\right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi^h(s_{t+1}|s_t) \left(U^h(c^h, s^{t+1})\right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}},$$

where $\frac{1}{1-\rho^h}$ represents the inter-temporal elasticity of substitution (IES) and $1 - \alpha^h$ the relative risk aversion of the agent.

4.2 Financial Markets and Collateral

At each date-event, agents can engage in security trading of two assets—the long-lived stock and a one-period bond. Agent h can buy $\theta^h(\sigma)$ shares of the long-lived asset at node σ for a price $q(\sigma)$. Importantly, agents can short-sell long-lived assets. We assume that short positions in the long-lived asset cannot be "naked", meaning that agents must cover their short sales by having borrowed the long-lived asset.[11] In addition to the long-lived asset, there is a single one-period bond available for trade; this bond is in zero net supply and its face value is one unit of the consumption good in

[11] Note that this assumption is in line with short-selling regulations in the EU and the US (see, e.g., `https://www.esma.europa.eu/regulation/trading/short-selling`).

the subsequent period. Agent h's (end-of-period) holding of this bond at date-event σ is denoted by $\phi^h(\sigma)$, and the price of the bond at this date-event by $p(\sigma)$. Agents can take up debt by shorting this bond.

The agents can default on short positions in the long-lived asset or the short-lived bond at any time without any utility penalties or loss of reputation. Therefore, to enter a short position in either of the two assets, agents must back up their promised payments by collateral. Since there are only two assets in the economy, an agent who shorts the bond must hold a long position in the long-lived asset as collateral. And vice versa, an agent who borrows by assuming a short position in the long-lived asset must hold a long position in the bond as collateral. In this paper, we assume that agents have to post enough collateral to prevent default in equilibrium.[12] Therefore, in any state next period, the value of the collateral asset has to exceed the short position that it covers. For borrowing with the bond against the long-lived asset, i.e. $\phi^h < 0 < \theta^h$, agents thus have to satisfy

$$-\phi^h(s^t) \leq \theta^h(s^t) \min_{s_{t+1}} \left\{ q(s^{t+1}) + d(s^{t+1}) \right\}. \tag{1}$$

This collateral constraint makes the bond risk-free by ensuring that a short-seller will never default on his or her promise. Similarly, for borrowing with the long-lived asset against the bond, i.e. $\theta^h < 0 < \phi^h$, agents have to satisfy[13]

$$-\theta^h(s^t) \max_{s_{t+1}} \left\{ q(s^{t+1}) + d(s^{t+1}) \right\} \leq \phi^h(s^t). \tag{2}$$

4.3 Re-use of Collateral

In our economy, when the long-lived asset is posted as collateral, the receiving agent has the right to reuse it for his or her own purposes, either using it as collateral for another transaction or (short-)selling it.[14] We denote the amount of collateral received by agent h by $\theta^h_{received}(s^t)$, and the amount of collateral reused by $\theta^h_{reused}(s^t)$. We furthermore assume that a regulatory agency can set a limit $\kappa(s^t) \in [0, 1]$ on the fraction of the received collateral that can be reused by agents. This leads to the following re-use constraint for agent h:

$$\theta^h_{reused}(s^t) \leq \kappa(s^t) \cdot \theta^h_{received}(s^t). \tag{3}$$

Using the collateral constraint (1) and the re-use constraint (3) we can now determine the maximum short position agents can assume. First of all, we need to determine how much collateral agent h receives, $\theta^h_{received}(s^t)$. The collateral constraint (1) that the other agent, $-h$, faces for a short position

[12] In Brumm et al. (2015a) this restriction (for short positions in the bond) is an equilibrium outcome: following Geanakoplos (1997) and Geanakoplos and Zame (2002), Brumm et al. (2015a) assume that, in principle, bonds with any collateral requirement may be traded in equilibrium, yet show that with moderate default costs only risk-free bonds are traded.

[13] Note that constraints (1) and (2) can alternatively be derived from the requirement that the portfolio payoff of all agents in all states must be non-negative.

[14] In financial markets, a significant amount of collateral is supplied including a transfer of title (e.g., in repo transactions) that implies that for the length of the transaction the counterparty receiving the collateral becomes the owner of the collateral. It is, therefore, free to use it as collateral in a different transaction or to sell it.

in the bond dictates how much collateral agent $-h$ has to post and thus how much agent h receives:

$$-\phi^{-h}(s^t) = \theta^h_{received}(s^t) \min_{s_{t+1}} \left\{q(s^{t+1}) + d(s^{t+1})\right\}, \text{ for } \phi^{-h}(s^t) \leq 0.$$

As there is no collateral posted when $\phi^{-h} \geq 0$, we get:

$$\theta^h_{received}(s^t) = \frac{\max\{0, -\phi^{-h}(s^t)\}}{\min_{s_{t+1}} \{q(s^{t+1}) + d(s^{t+1})\}}. \tag{4}$$

The lender, agent h, can reuse a portion $\kappa(s^t)$ of this collateral for his own purposes. If he uses it for short-selling, we have $\theta^h(s^t) = -\theta^h_{reused}(s^t)$. Combined with (3) and (4), we obtain a lower bound on the position of agent h in the long-lived asset,

$$\theta^h(s^t) \geq -\kappa(s^t) \frac{\max\{0, -\phi^{-h}(s^t)\}}{\min_{s_{t+1}} \{q(s^{t+1}) + d(s^{t+1})\}}. \tag{5}$$

Observe that for $\phi^{-h}(s^t) < 0$ (and thus $\phi^h(s^t) > 0$) the lower bound is negative and agent h is permitted to short the long-lived asset. Essentially he sells a portion of the collateral owned by agent $-h$ but pledged to him. Note that the re-use constraint (5) is also valid for $\phi^h(s^t) \leq 0$ and then reduces to $\theta^h(s^t) \geq 0$. Without having a positive bond position for which collateral is posted, an agent cannot short the the long-lived asset. Finally, note that setting the re-use parameter $\kappa(s^t)$ equal to zero would lead to a situation where not only is re-use ruled out but also short-selling is banned completely.

4.4 Financial-Markets Equilibrium with Collateral

We are now in the position to formally define a financial-markets equilibrium. Equilibrium values of a variable x are denoted by $\bar{x}$. Note that we have written all three constraints as providing lower bounds on the future (minimal or maximal) value of the long-lived asset.

Definition 1 *A financial-markets equilibrium for an economy with regulated re-use limits,* $(\kappa(\sigma))_{\sigma\in\Sigma}$*, initial shock* s_0*, and initial asset holdings* $(\theta^h(s^{-1}))_{h\in\mathcal{H}}$ *is a collection of agents' portfolio holdings, consumption allocations and security prices,* $\left(\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\right)_{h\in\mathcal{H}}; \bar{q}(\sigma), \bar{p}(\sigma)\right)_{\sigma\in\Sigma}$*, satisfying the following conditions:*

(1) Markets clear:

$$\sum_{h\in\mathcal{H}} \bar{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h\in\mathcal{H}} \bar{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.$$

(2) For each agent h*, the choices* $\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\right)$ *solve the agent's utility maximization problem,*

$$\begin{aligned}
&\max_{\theta^h, \phi^h, c^h} U_h(c) \quad \text{s.t.} \quad \text{for all } s^t \in \Sigma \\
c(s^t) + \theta^h(s^t)\bar{q}(s^t) + \phi^h(s^t)\bar{p}(s^t) \quad &= \quad e^h(s^t) + \phi(s^{t-1}) + \theta^h(s^{t-1})\left(\bar{q}(s^t) + d(s^t)\right), \\
-\phi^h(s^t) \quad &\leq \quad \theta^h(s^t) \min_{s_{t+1}} \left\{q(s^{t+1}) + d(s^{t+1})\right\}, \\
-\phi^h(s^t) \quad &\leq \quad \theta^h(s^t) \max_{s_{t+1}} \left\{q(s^{t+1}) + d(s^{t+1})\right\}, \\
-\kappa(s^t) \max\{0, -\bar{\phi}^{-h}(s^t)\} \quad &\leq \quad \theta^h(s^t) \min_{s_{t+1}} \left\{q(s^{t+1}) + d(s^{t+1})\right\}.
\end{aligned}$$

4.5 The Calibration

We determine the parameters of the model in a two-step procedure. The first set of parameters, mainly governing the exogenous endowment process, is taken from external sources, while the second set of parameters, mainly relating to preferences and beliefs, is chosen such that simulations of the model match moments of US asset-pricing data.

We consider a growth economy with stochastic growth rates. The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$, which only depends on the new shock $s_{t+1} \in \mathcal{S}$, thus

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t \in \Sigma$. There are four different realizations for $g(s_t)$. We declare the first to be a "disaster". We calibrate the disaster shock based on data from Barro and Ursúa (2008). A disaster is defined as a drop in aggregate consumption of more than 15 percent, which has a probability of 2.2 percent and an average size of 28 percent (see Table 10 in Barro and Ursúa (2008)). Following Barro (2009), we choose transition probabilities such that the four exogenous shocks are i.i.d. The non-disaster shocks are then calibrated such that their average growth rate is 2 percent and their standard deviation matches the data on typical business cycle fluctuations, which have a standard deviation of about 2 percent. Table I provides the resulting growth rates and their probabilities. Because of their respective sizes, we refer to these realizations as follows: disaster, recession, normal times, and boom.

Table I: Growth rates and their probabilities

	Disaster	Recession	Normal Times	Boom
Growth rate g	0.72	0.96	1.02	1.08
Probability $\pi(g)$	0.022	0.054	0.870	0.054

The table reports the four possible realizations for the growth rate of aggregate endowments and their respective probabilities.

We assume that the Lucas tree pays dividends, which are, for simplicity, proportional to aggregate endowments, $d(s^t) = \mathfrak{d}\bar{e}(s^t)$, $\mathfrak{d} \geq 0$. In our baseline calibration, the dividend share is 10 percent—that is, $\mathfrak{d} = 0.10$—roughly equal to the value of dividends relative to the total income of stockholders in the US. The remaining 90 percent share of aggregate income is received by the two agents as endowments. We abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks in a model with only two types of agents. Agent 1's endowment is given by

$$e^1(s^t) = \tilde{e}^1 \left(\bar{e}(s^t) - d(s^t) \right),$$

where $\tilde{e}^1$ parametrizes the share of non-dividend endowments that goes to agent 1. This parameter is determined in the calibration procedure summarized in Tables II and III below.

We assume that agents disagree about the likelihood of a disaster. Given the assumption that a disaster occurs about once every 50 years this seems a reasonable assumption. In order to keep the number of parameters as small as possible we assume that the agents' beliefs deviate from the objective probabilities in opposite directions, agent 1 being optimistic and agent 2 being pessimistic. Agent 1 believes that the probability of a disaster is $(1-\delta^d)$ times its objective probability, while agent 2 believes it is $(1+\delta^d)$ times its objective probability. The remaining probability weight is distributed among the other growth realizations according to their objective probability weights. We calibrate the disagreement parameter δ^d as described below.

Recall that the agents have recursive utility functions (Epstein and Zin (1989)) with parameters ρ^h and α^h where $1/(1-\rho^h)$ determines the inter-temporal elasticity of substitution (IES) and $1-\alpha^h$ determines the relative risk aversion of the agent. We assume that both agents have an identical IES of 0.5. This value lies in the middle of the empirical estimates from the micro consumption literature (see, e.g., Attanasio and Weber (1993)), and is also very commonly used in the macro and public finance literature (note that it implies a coefficient of relative risk aversion of 2 with standard CRRA preferences). In finance (e.g., Barro (2009)), it is often assumed that the IES is above 1. In a sensitivity analysis in Appendix A we repeat the analysis assuming that agents have an identical IES of 1.5.

We assume that agents have an identical time discount factor, β, but differ in their risk aversion (in addition to the disagreement over the disaster probability). We choose the disagreement parameter, δ^d, the risk aversion parameters, α^1 and α^2, the income share of agent 1, $\tilde{e}^1$, and the discount factor of both agents, β, to match key asset-pricing moments. These parameters are chosen to match the first and second moments of risk-free and risky returns as well as the price-dividend ratio for US data from 1930 to 2008 as reported in Beeler and Campbell (2012). We match these data moments for the case of unlimited re-use as this case best approximates the general treatment of re-use in real-world regulatory frameworks where constraints exist only for some forms of re-use in some jurisdictions (see Sec. 2). In the model, there is no simple one-to-one correspondence between a parameter and a moment. For example, as we explain in detail in the next section the volatility of asset returns is jointly determined by differences in beliefs, differences in risk aversion, and the income share of agent 1. We do not attempt to match the moments exactly, but rather try to find "simple" (e.g. only one or two significant digits) values for the parameters to match the moments approximately.

Table II: Calibration targets

Target	Calibrated model	Data
Mean equity return (in %)	5.8	5.5
Mean risk-free rate (in %)	0.39	0.56
STD risky returns (in %)	19	20
STD risk-free returns (in %)	3.3	2.9
Log price-dividend ratio	3.4	3.4

The table states the empirical moments reported in Beeler and Campbell (2012) and the respective values generated by our calibrated model (with two significant digits). The abbreviation "STD" stands for "standard deviation".

Table II states the moments from Beeler and Campbell (2012) that we target and the respective

Table III: Calibrated parameters

Parameter	Calibration
Risk aversion coefficient agent 1, $1-\alpha^1$	3
Risk aversion coefficient agent 2, $1-\alpha^2$	7
Discount factor of both agents, β	0.94
Endowment share agent 1, $\tilde{e}^1$	0.1
Disagreement on disaster, δ^d	0.8

The table states the parameter values of our calibrated model.

values generated by our calibrated model. The baseline calibration produces realistic first and second moments of equity returns and the risk-free rate.[15] Table III summarizes the parameter values of the baseline calibration. The values for risk aversion lie well within the range of values that are realistic and they are consistent with lab experiments. The disagreement is rather large—however, the resulting probabilities of the disaster state do not seem unrealistic: one agent believes the state is very unlikely with a probability just above 0.4 percent while the other agent is overly pessimistic and believes that it has a probability of almost 4 percent. The less risk-averse agent is also the optimistic agent. This model feature is consistent with evidence from surveys that document a positive correlation between pessimism and risk aversion (see, e.g., Dohmen et al. (2017)).

5 The Impact of Re-use on Equilibrium Dynamics

In this section we demonstrate that the option to reuse collateral increases leverage in the economy and has large effects on the volatility of asset returns. Before we begin our analysis, it serves us well to recall the two-period model from Section 3.

In our discussion of the two-period model, we learned that in an economy with re-use-constrained agents, an increase in the re-use parameter κ enables the agents to hold portfolios with higher leverage. Specifically, the pessimistic and more risk-averse agent 2 can assume larger short positions in the stock to finance larger investments in the safe bond. Conversely, this enables the more optimistic and less risk-averse agent 1 to buy larger stock positions, which he finances by larger debt positions in the bond. In our analysis below, we observe that increasing the re-use parameter has the same effect on equilibrium portfolios in the infinite-horizon economy. However, the more leveraged portfolios of the two agents have strong additional effects in the dynamic setting, which cannot be captured in a two-period model. The exogenous growth process including disaster and boom states leads to large changes in the wealth distribution over time since their leveraged portfolios make both agents' financial wealth susceptible to the exogenous shocks. In a disaster state, agent 1's wealth share declines dramatically. In a boom state, particular after a long series of normal growth states, agent 2's wealth share shrinks considerably. We now document that such changes in the wealth

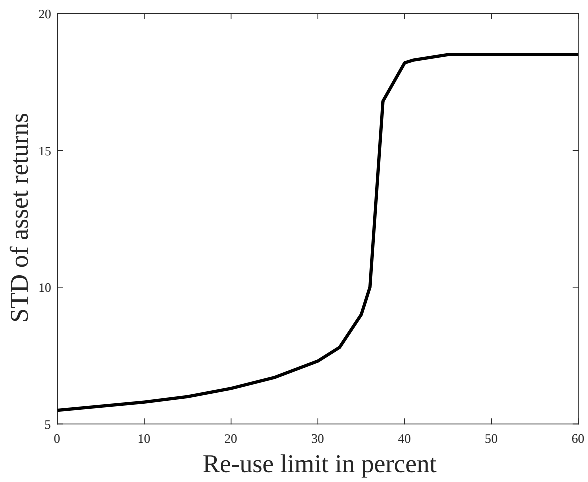
[15] We show in Appendix B that it is not essential for generating moments of such magnitude that disasters actually occur in simulations.

distribution matter quantitatively and lead to large price effects.

5.1 Leverage and Volatility

We begin our analysis by examining how the standard deviation of stock returns depends on the re-use parameter κ. Recall that for $\kappa = 0$ re-use is not allowed and for $\kappa = 1$ there are no constraints on re-use. Figure IV plots the stock return volatility as a function of the re-use parameter. The

Figure IV: Stock return volatility as a function of the re-use parameter

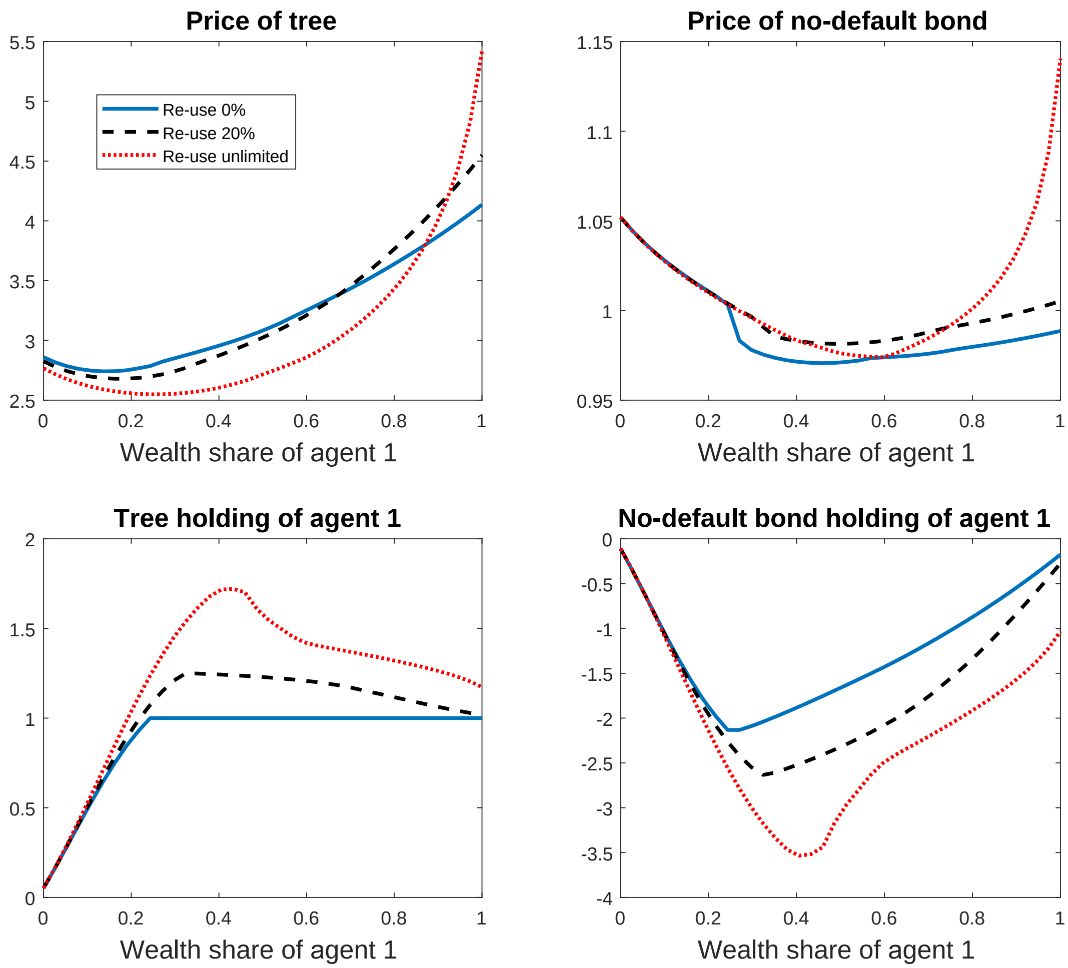


The figure plots the standard deviation (STD) of the stock return as a function of the re-use parameter κ.

stock return volatility increases monotonically as the re-use parameter increases. Once the re-use parameter reaches a value of about 45 percent, the re-use constraint is never binding along simulated equilibrium paths; hence, any further relaxation of the constraint does not affect the stock's return volatility. We observe not only a qualitative but also a strong quantitative effect of κ on the volatility. For $\kappa \geq 0.45$ the stock return volatility is more than three times as large as the volatility in an economy without re-use, $\kappa = 0$. A remarkably strong increase in the volatility occurs when the re-use parameter increases from 30 to 40 percent. This moderate change of κ more than doubles the volatility.

To obtain a clear understanding of the documented reaction of the volatility of returns in response to changes in the re-use parameter κ, we next work out the economic mechanism driving the wealth and asset-pricing dynamics in the economy using the computed policy and pricing functions. Figure V depicts equilibrium asset prices and asset holdings as a function of the endogenous state variable, agent 1's share of financial wealth. Just as in the two-period model, agent 1—the agent with the low risk aversion parameter and optimistic beliefs—is the natural buyer of the risky stock and borrows in the safe bond to finance his stock investments. Agent 2—the agent with the high risk aversion and pessimistic beliefs—has a strong desire to insure against bad shocks and is thus willing to provide financing to the other agent backed by collateral. These equilibrium portfolios are clearly visible in

Figure V: Price and policy functions



The figure plots the price and policy functions for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$, as a function of the endogenous state variable, the wealth share of agent 1. The top row shows the stock price (left) and the bond price (right). The bottom row displays agent 1's holdings of the stock (left) and of the bond (right). Note that these functions are identical across all date-events since shocks in the economy are i.i.d..

the two policy functions in the bottom row of Figure V. For all possible values of the endogenous state variable (the wealth share of agent 1), agent 1 holds a long position in the risky stock and a short position in the riskless bond. This portfolio structure is present for all possible values of the re-use parameter. The policy functions also clearly show the impact of the re-use parameter on the agents' portfolio positions. In the economy without re-use, $\kappa = 0$, agent 1's stock position is bounded above by the aggregate supply of one share. Once his wealth share increases above 0.2, he holds the entire stock; his short position in the bond is then decreasing as his wealth share increases further. If re-use is permitted, then agent 1's stock position is no longer bounded above by the aggregate supply. For positive values of κ, agent 2 can now reuse the collateral (the stock) that she receives for lending to agent 1 via her bond purchases; agent 2 now sells a portion of the collateral to agent 1. As a result, agent 1's position now exceeds the aggregate supply of one share. This agent finances the additional stock purchases with additional borrowing in the bond. Thus, his short position in the bond now becomes much larger than in the economy without re-use.

It is worthwhile pointing out that, starting from the economy without re-use, an increase in the re-use parameter to $\kappa = 0.2$ has a comparatively modest effect on the equilibrium portfolios while moving from $\kappa = 0.2$ to $\kappa \geq 0.45$ leads to much larger effects on the portfolios. The agents' holdings become much more leveraged. This increased leverage leads to a substantial increase in the volatility of the wealth distribution, which, in turn, contributes to the drastic increase in the stock return volatility—as we discuss in detail in Section 5.3 below.

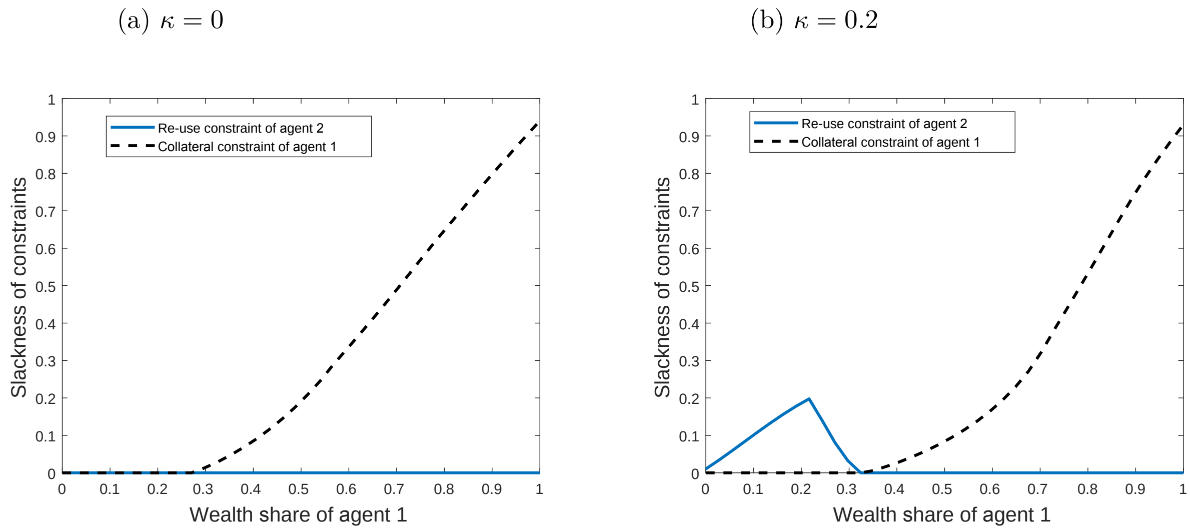
The top row of Figure V shows that an increase of the re-use parameter κ affects not only the portfolio holdings but also the equilibrium asset prices. Again, we observe modest changes when we compare the economy with $\kappa = 0$ to the economy with $\kappa = 0.2$. In contrast, for $\kappa \geq 0.45$ both price functions are very steep in the right part of the state space, where the optimistic agent 1 is rich. As a result, changes in the wealth distribution lead to much larger price fluctuations than in a tightly constrained economy. Such price effects also contribute to the behavior of the stock return volatility displayed in Figure IV.

While we simply refer to "agent 1" and "agent 2" throughout most of the paper, it is worthwhile to recall that there is a continuum of each type of agent in the economy. When an agent (of type 2) receives an asset as collateral (from a type 1 agent) and sells short this asset to a third agent, then this agent (while of type 1) is not identical with the agent who posted that collateral. In this sense there are collateral chains in our model, which lead (as we discuss extensively) to higher leverage and volatility. Of course, with more heterogeneity and/or default in the model, a deeper analysis of these collateral chains and of how they unfold after large shocks would be possible, yet this is beyond the scope of the current paper.

5.2 A Tale of Two Constraints

The policy functions in the bottom row of Figure V are non-monotone with clearly visible extrema. To understand this non-monotone behavior of the policy functions, we next turn to the constraints in the agents' utility maximization problems; two constraints play important roles in equilibrium. Figure VI displays the slack in the collateral constraint of agent 1 and the slack in the re-use constraint of agent 2 for the economy without re-use (panel (a)) and for the economy with limited re-use, $\kappa = 0.2$ (panel (b)).

Figure VI: Slack in constraints



The figure shows the slack in the re-use constraint (3) of agent 2 and the collateral constraint (1) of agent 1, respectively. For $\kappa = 0$, agent 2's re-use constraint (3) reduces to $\theta^2_{reused}(s^t) \leq 0$ and so the slack is trivially zero on the entire state space. For $\kappa = 0.2$, agent 2's re-use constraint (3) is $\theta^2_{reused}(s^t) \leq 0.2 \cdot \theta^2_{received}(s^t)$. The slack in this constraint peaks at a level of 0.2 when agent 1 holds exactly the unit net supply of the stock.

In the economy without re-use, $\kappa = 0$, the re-use constraint (3) is $\theta^h_{reused}(s^t) \leq 0$ and thus is—trivially—always binding. This constraint implies a short-selling constraint, $\theta^2(s^t) \geq 0$, for agent 2, which is binding for agent 2 whenever agent 1 holds the entire stock. This case occurs whenever the wealth share of agent 1 exceeds a threshold of about 0.27; see also Figure V. As agent 1's wealth share increases beyond a level of 0.27, he cannot buy more of the stock but only reduce his debt in the bond. Therefore, his collateral constraint (1) is not binding in this region of the state space. On the contrary, when agent 1 has a wealth share below the threshold of 0.27, he buys as much stock as permissible under the collateral constraint (1) and so this constraint is binding.

In the economy with limited re-use, $\kappa = 0.2$, the collateral constraint of agent 1 is binding when his wealth share is below a threshold of about 0.32. In this region of the state space, agent 1 buys as many shares of the stock as permissible under the collateral constraint (1). For wealth shares above this threshold, the re-use constraint of agent 2 becomes binding and then agent 1 can no longer increase his stock holding. As agent 1's wealth share increases for levels above 0.32, he can only reduce his debt in the bond. But then the re-use constraint (3) forces agent 2 to reuse less, which, in turn, means that agent 1 must reduce his long stock position and, as a consequence, reduce his bond debt even further. This is reflected in the fact that agent 1's bond-holding function (in Figure V) is increasing—that is, his debt is decreasing—in the region where agent 2's re-use constraint binds. Nevertheless, agent 1 remains leveraged on the entire domain of the endogenous state variable.

As a result of the two key constraints, leverage peaks at the wealth share where both constraints are binding. These threshold points lead to the global maxima of agent 1's stock-holding function and the related global minima of his bond-holding function in Figure V. As agent 1's wealth share increases beyond these points, his long position in the stock and his short position in the bond become decreasing in the endogenous state variable.

5.3 Equilibrium Dynamics and the Wealth Distribution

We continue our analysis by taking a look at equilibrium dynamics. Figure VII shows simulated paths of key economic variables for an economy with unconstrained re-use, $\kappa = 1$. The bottom graph in the figure indicates that even over only 100 time periods the endogenous state variable, the wealth share of agent 1, may fluctuate between values close to its minimum, zero, and its maximum, 1. Put differently, the economy with unconstrained re-use, $\kappa = 1$, exhibits considerable volatility in its wealth distribution. In the vast majority of periods agent 1's wealth share exceeds 0.2. And so, according to the graph at the bottom left of Figure V, his stock holding exceeds 1. This fact is also apparent in the graph displaying agent 2's re-use of collateral. We observe that her re-use is positive for the vast majority of periods. Throughout the entire simulation, agent 1 has considerable debt via his short position in the bond.

When the economy is hit by a disaster shock, the leveraged agent 1 loses financial wealth. In the simulation in Figure VII, disaster shocks occur in periods 22, 33, 69, and 84. Each time, agent 1's wealth share drops dramatically to values below 0.2. As a result, the collateral constraint becomes binding and forces him to reduce consumption, to reduce his stock holding, and to reduce his short position in the bond. In fact, he must reduce his stock position below 1. And so, for a few time periods after each disaster, agent 2 must be long in the stock; her re-use is briefly zero.

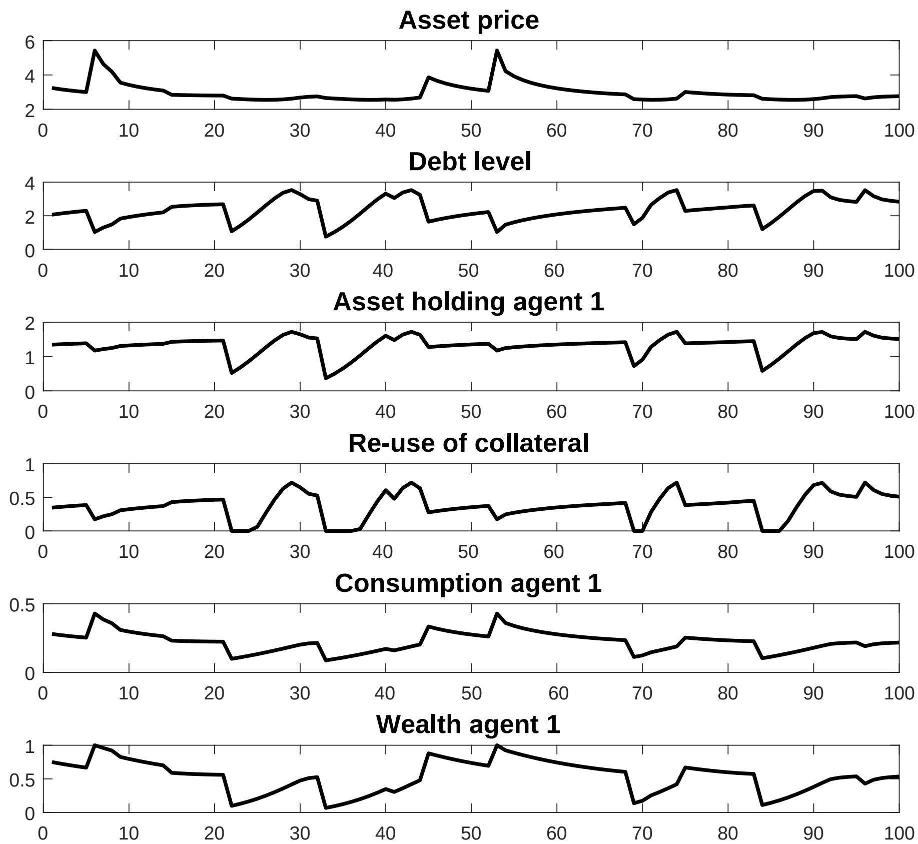
Long episodes of normal and boom shocks move the endogenous state variable in the opposite direction. Agent 1's wealth share increases and may, in fact, hit one—that is, after a series of good shocks agent 1 holds the entire wealth in the economy. The good shocks greatly increase the stock price and thereby the repayment obligations of agent 2, who is re-using the stock for short sales. As a result, her wealth share tends to zero during such episodes.

Figure VII indicates that in our calibration the (realistically) high volatility of stock returns is driven to a very large extent by a high volatility in the wealth distribution. As agent 1 becomes very rich, the stock price spikes. One reason for this effect is that due to the re-use constraint the agent remains leveraged even when he is very rich. Another reason is, of course, that we assume a low elasticity of inter-temporal substitution, which leads to higher asset prices if one agent is rich.

We finish our discussion in this section with a comparison of asset-pricing moments and other key economic variables across economies with different values for the re-use parameter. Table IV reports simulation statistics for three economies: an economy in which re-use is banned entirely, $\kappa = 0$, an economy with an intermediate level of re-use, $\kappa = 0.2$, and an economy with unconstrained re-use, $\kappa = 1$. For our discussion of these statistics, we also include Figure VIII, which displays the tree price functions together with densities of the endogenous wealth distribution for the three economies.

The results in Table IV reflect our description of the economic mechanisms that are present

Figure VII: Simulated paths of equilibrium quantities



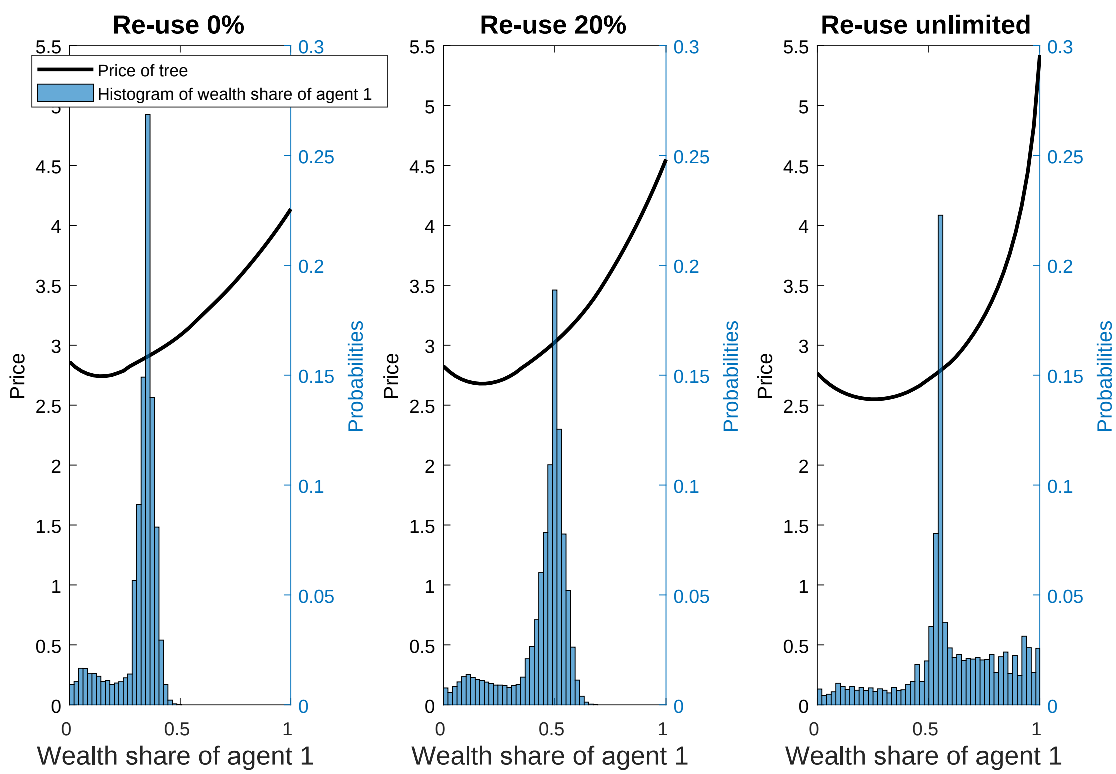
The figure plots simulated paths of six different equilibrium quantities over 100 periods for the economy with unconstrained re-use, $\kappa = 1$. Disaster shocks occurred in periods 22, 33, 69, and 84. The top graph, entitled "Asset Price", shows a simulation path for the stock price. The graph entitled 'Debt level" shows agent 1's short position in the safe bond. The graph entitled "Re-use of collateral" shows the path of agent 2's short position (in absolute terms) in the stock, which is identical to $\max\{0, \theta^1(s^t) - 1\}$.

Table IV: Simulation statistics for the model without re-use and with re-use

	no re-use ($\kappa = 0$)	some re-use ($\kappa = 0.2$)	free re-use ($\kappa = 1$)
mean wealth, agent 1 (in %)	31	44	60
STD wealth, agent 1 (in %)	9.2	13	22
mean re-use rate (in %)	0.0	17	32
mean bond holding, agent 1	-1.9	-2.2	-2.3
mean equity return (in %)	4.9	4.9	5.8
mean risk-free rate (in %)	1.8	1.3	0.39
STD equity returns (in %)	5.5	6.3	19
STD risk-free returns (in %)	1.9	1.4	3.3

The table reports simulation statistics for our economic model for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$. The re-use constraint is never binding for $\kappa \geq 0.45$. Therefore, the simulation statistics in the column for free re-use ($\kappa = 1$) are representative for all $\kappa \in [0.45, 1]$.

Figure VIII: Price of tree and histogram of agent 1's wealth shares



The figure plots the price function of the tree for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$. In addition, the three graphs show a histogram of agent 1's wealth shares during a long simulation.

in the dynamic economy. The first row of the table shows that agent 1's average wealth share in the economy is increasing in the re-use parameter. At the same time, the support of the wealth distribution is spreading out further, leading to more volatile wealth shares for the two agents, as one can nicely see in Figure VIII. The average re-use rate, defined as reused collateral divided by collateral received that is eligible for re-use, increases strongly with the re-use parameter κ. As a consequence, agent 1 holds, on average, a much larger short position in the bond as κ increases. The portfolios of both agents exhibit more leverage as the re-use parameter increases.

We observed in Figure IV at the beginning of this section that the volatility of the stock returns is first modestly and then drastically increasing in κ. This volatility is more than three times as large in the economy with free re-use (18.5%) than in the economy without re-use (5.5%). The average equity excess return is increasing due to a decreasing risk-free rate and a modestly increasing equity return. The volatility of the risk-free rate is non-monotone, which is in line with the bond price function in the upper right graph of Figure V.

Table V reports the resulting Sharpe ratios. Since the equity return volatility grows much faster than the equity premium when the re-use parameter κ increases, we observe that the realized Sharpe ratios in the economies with constrained re-use are much larger than in the economy with free re-use.[16] Table V also reports the subjective Sharpe ratios of the two agents, calculated with their subjective beliefs. As one would expect, the subjective Sharpe ratios of the optimistic and less risk-averse agent are always much larger than those of the pessimistic and more risk-averse agent, explaining their differing demand for the stock.

Table V: Simulated Sharpe ratios

	No re-use ($\kappa = 0$)	Some re-use ($\kappa = 0.2$)	Free re-use ($\kappa = 1$)
Realized Sharpe ratio	0.55	0.57	0.32
Subjective Sharpe ratio, agent 1	1.06	1.05	0.39
Subjective Sharpe ratio, agent 2	0.35	0.38	0.27

The table reports simulated Sharpe ratios for our economic model for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$. The subjective Sharpe ratios are the averages of the agents' one-period-ahead expected Sharpe ratios based on their subjective probabilities. The realized Sharpe ratio is the actual average ratio during a long simulation.

6 Welfare Implications of Re-use Limits

We now analyze the welfare implications of limiting the re-use of collateral. To do so, we first have to take a stand on how to measure welfare in the presence of heterogeneous beliefs. One can, of course, take the extreme position that probabilities are part of a mathematical representation of preferences and have no meaning outside the context of this representation. If one follows this line of thought, the only reasonable way to measure agents' welfare is based on their subjective

[16] Note that the Sharpe ratios for the constrained economies can only be so large because we do not recalibrate the model for the counterfactuals with $\kappa < 1$.

beliefs—and, measured like this, regulation of re-use makes everybody worse off.[17] Alternatively, as Gilboa et al. (2014) point out, "Savage's [...] derivation of subjective expected utility maximization [...] is consistent with a view of probabilities and utilities as conceptually different. The view that probabilities are not empty, theoretical constructs runs throughout economics." In the recent literature, several alternative definitions of Pareto optimality under differences in beliefs have been proposed (see, for example, Brunnermeier et al. (2014) or Gilboa et al. (2014) and the references therein). Following Brunnermeier et al. (2014), we say that an allocation A is (belief-neutral) Pareto-better than allocation B if under all beliefs in the convex hull of agents' subjective probabilities (all "reasonable" beliefs) the allocation makes both agents better off (perhaps after a redistribution). It turns out that under this interpretation our welfare analysis reveals a startling and robust result: the impact of tightening the re-use constraint on welfare is non-monotone and there are levels of regulation that are welfare improving, both upon no re-use and upon unrestricted re-use.

We consider unanticipated changes in regulation that occur between two periods. Our starting point is always the setting with no re-use limits, $\kappa = 1$. The regulator then introduces a re-use limit with a specific $\kappa < 1$. It is important to note that the change in welfare depends both on the wealth distribution in the period before the regulatory change and on the exogenous state in the period when the regulation first applies. The wealth distribution matters because it affects both asset prices and agents' portfolio choices. As we have seen in the previous section, different levels of the re-use limit imply different price and policy functions. The exogenous state matters because it may greatly affect the wealth distribution. In light of these observations, we need to decide for which values of the endogenous and exogenous state we evaluate the change in welfare. We believe that the most reasonable point of departure is the median of the ergodic wealth distribution of the unregulated economy and the exogenous state 3, which is the mode of the distribution of shocks and represents the normal growth state. We refer to this starting point as the "benchmark economy" in our discussion below. To demonstrate that the choice of starting point matters only quantitatively but not qualitatively, we also verify that our key results hold for the 10th and the 90th percentile of the ergodic wealth distribution.

As we pointed out above, changes in the re-use limit, κ, typically have large effects on the price of the tree. Since the tree is held entirely by agent 1, this price change results in large welfare effects that are due to a redistribution of financial wealth. In our analysis below we report "compensated welfare gains" in the sense that we compensate agent 1 by means of a (one-time) transfer to or from agent 2, the size of the transfer being chosen such that the welfare of agent 1 remains as if there were no change in re-use limits.

We first examine welfare effects in our baseline calibration from Section 4.5 above. In the appendix we show that our results are robust and also hold for other calibrations of the model. We follow Brunnermeier et al. (2014) in that we show that regulation can lead to allocations that are Pareto-better under all reasonable beliefs. We begin our analysis with the simpler case where we only use the true beliefs to evaluate welfare. It is useful to discuss this case in detail because it makes clear

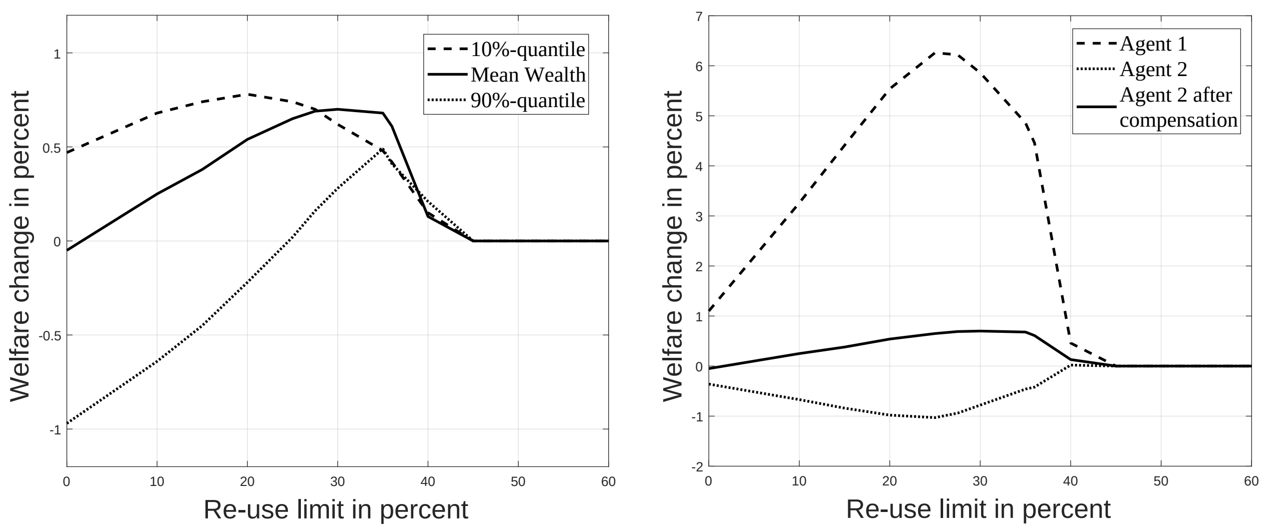
[17] Although the first welfare theorem does not hold in our incomplete-markets economy, the welfare losses from tightening binding constraints are of the first order and dominate all other possible effects.

how regulation can improve welfare. We then turn to the full analysis and evaluate welfare under all reasonable beliefs.

6.1 Welfare Analysis under True Beliefs

The left graph in Figure IX plots the welfare effects for agent 2 of an unanticipated change in the re-use regulation when the welfare for agent 1 is kept constant by means of transfers. The interior maximum is clearly visible. The right graph in Figure IX plots the individual welfare changes for the two agents and also reports the post-compensation change for agent 2 for the benchmark economy. All welfare changes are exactly zero for $\kappa \in [0.45, 1]$. For such large values of the re-use parameter,

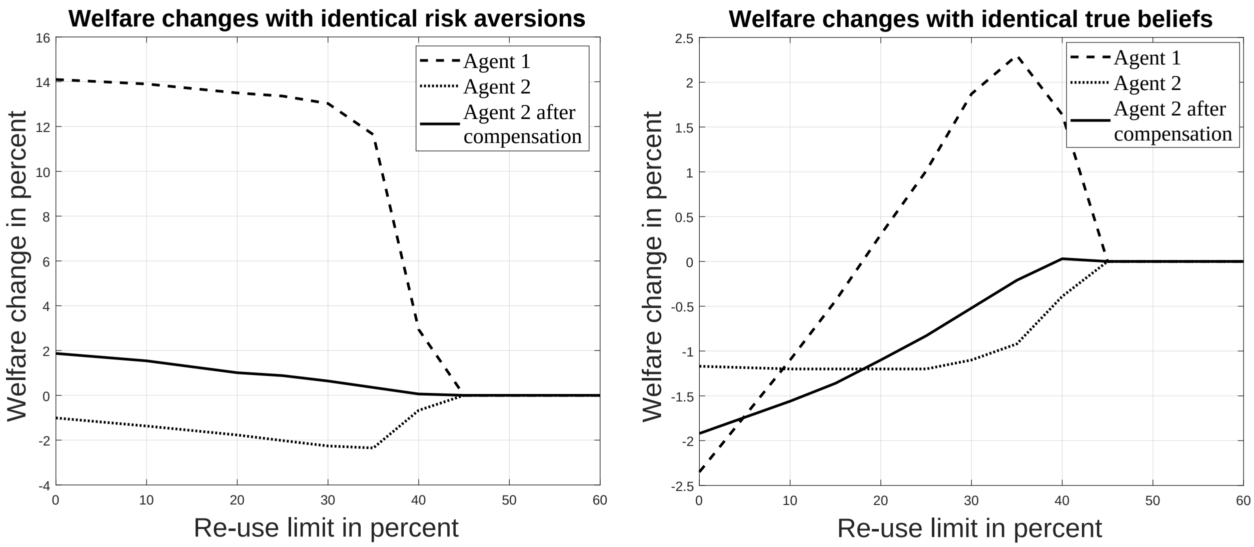
Figure IX: Welfare changes as a function of the re-use parameter



In the left graph, the solid line shows the welfare change for agent 2 after compensating agent 1 for the impact of the regulatory change from $\kappa = 1$ to the level of κ (in percent) on the horizontal axis. The benchmark economy for the comparison is an unregulated economy in a time period when agent 1's wealth is equal to the mean of the ergodic distribution; also, the economy enters state 3 in the subsequent period. The other two lines report the corresponding welfare changes for an unregulated economy in a time period when agent 1's wealth is equal to the 10th and 90th percentile, respectively. The right graph shows the welfare changes for both agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis and also reports the post-compensation change for agent 2 for the benchmark economy.

the re-use constraint is never binding in long simulations. The equilibrium is identical to that of an economy with free re-use, and changing κ in this region does not affect equilibrium outcomes. For values of the re-use parameter in the interval $[0.4, 0.45]$ (approximately), regulation leads to very small positive welfare effects for both agents. The price effects are very small and both agents are slightly better off than without regulation. More interestingly, for stricter regulation with smaller

Figure X: Disentangling the welfare effects



Both graphs show welfare changes for the two agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. The left graph shows the welfare changes for a model in which both agents have an identical risk aversion of 3 and the original heterogeneous beliefs. The right graph shows the welfare changes for a model in which the agents have identical correct beliefs and the original heterogeneous levels of risk aversion.

values of the re-use parameter welfare gains are substantial. Because of a large price effect (the price of the tree increases) agent 1 gains and agent 2 loses welfare. However, after compensation of agent 1, the net effect on agent 2 turns into a positive welfare change.

Why is the impact of regulation on welfare non-monotone? Two counteracting economic forces are at play. First, the ability to reuse allows for more risk sharing in the economy. This effect is generally beneficial for welfare given the agents' heterogeneity in risk aversion. Second, the heterogeneity in agents' beliefs triggers agents to build up leveraged positions beyond what is needed to optimally share risks (under the true beliefs). As the ability to reuse collateral allows agents to build up this leverage, limiting re-use has the potential to steer agents' choices closer to levels that are optimal under the true beliefs.

To demonstrate these two opposing economic forces, we disentangle the effects of heterogeneous risk aversion and heterogeneous beliefs in two separate experiments. In the first experiment, we set the coefficient of risk aversion of both agents to 3 and maintain their heterogeneous beliefs. The left graph in Figure X plots the individual welfare changes for the two agents in this experiment. The welfare-maximizing policy (after compensation and according to the true beliefs) is a policy prohibiting re-use. This result is not surprising. The agents have heterogeneous beliefs, which are both incorrect. Agent 1 is too optimistic and agent 2 is too pessimistic compared to the objective (and correct) beliefs. The availability of free re-use enables the agents to build up leveraged positions that are suboptimal (according to the true beliefs). We also note that the welfare gain of agent 2, after compensation of agent 1, is almost linear, if anything slightly concave, for $\kappa \in [0, 0.4]$.

In a second experiment, we set the beliefs of both agents to the true beliefs and maintain their original levels of risk aversion. The right graph in Figure X plots the individual welfare changes for the two agents in this experiment. Due to their heterogeneous levels of risk aversion but identical true beliefs, the agents trade solely for the purpose of risk sharing. A decrease of the re-use parameter to below 45 percent restricts the agents' ability to trade and decreases the welfare of the more risk-averse agent, agent 2. While the less risk-averse agent 1 gains welfare for some intermediate values of κ, he also loses welfare once the re-use parameter falls below 18 percent. Moreover, we find that the welfare of agent 2 after compensation of agent 1 is increasing in the re-use limit for $\kappa \in [0, 0.4]$, showing a slightly convex shape for $\kappa \in [0, 0.35]$.

In sum, for $\kappa \in [0, 0.4]$, we observe a weakly concave decreasing welfare of agent 2 after compensation of agent 1 in the first experiment and a slightly convex increasing welfare of agent 2 after compensation of agent 1 in the second experiment. Combining these insights, it is not surprising to find the hump-shaped impact of the re-use limit on welfare in the original setting with heterogeneous beliefs and heterogeneous risk aversion.

6.2 Welfare Analysis under All Reasonable Beliefs

Until now we have measured welfare gains using the agents' value functions under true beliefs. In this section we want to argue that regulation improves welfare under all reasonable beliefs; that is to say, all beliefs that are convex combinations of the two agents' beliefs—the true beliefs being just one case of many. This makes the argument much more convincing, as it is quite plausible to assume that agents would agree that some belief in the convex hull of their subjective beliefs describes the actual law of motion and that agents would therefore ex ante agree to proposed regulation that improves welfare under all such beliefs. Similarly, while it might be unrealistic to assume that the regulator's beliefs are identical to the true beliefs, it is quite plausible to assume that the regulator has beliefs that are reasonable in the above specified sense.

To present the results we first examine welfare changes under five alternative sets of beliefs. Two natural alternatives are the individual beliefs of the two agents. In addition, we consider beliefs that are the average of each of the agent's beliefs and the true beliefs. For each of these five alternatives, we compute again the maximal welfare change of agent 2 after compensation of agent 1. Table VI shows the results of this first exercise, reporting the location of the peak of the welfare function in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at the peak (in parentheses). The entry under "benchmark" denotes the results under true beliefs, the entry under "average 1" denotes the results for beliefs that are the average of agent 1's and the true beliefs, and so on. For all five beliefs, the welfare measure has an interior maximum. The location of the peak changes only slightly in response to changes in the beliefs, while the accompanying maximal welfare gains change considerable in response to changes in the beliefs. In all of the cases, (compensated) welfare gains are positive. It turns out that they are

Table VI: Sensitivity to choice of the beliefs used to evaluate welfare

Agent 1	Average 1	Benchmark	Average 2	Agent 2
27.5 (1.75)	27.5 (1.25)	30 (0.70)	35 (0.40)	35 (0.20)

The table reports the location of the peak of the welfare function in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at this location (in parentheses) for our economic model for the agents' value functions under different beliefs. The entry under "Benchmark" denotes the welfare change under true beliefs, the entry under "Average 1" denotes the welfare change for beliefs that are the average of agent 1's and the true beliefs, and so on.

much larger if welfare is evaluated under the beliefs of agent 1 (with a value of 1.75 percent) than when they are evaluated under the beliefs of agent 2 (with a value of 0.20 percent). This finding is consistent with the fact that agent 2 actually reuses the tree for short sales. The re-use constraint is binding for her, and so, if utility is evaluated under her beliefs, her welfare loss from regulation before compensation is rather large.

We repeated this exercise for another 10 convex combinations of the two agents' beliefs and observed that under all these convex combination of agents' beliefs regulating re-use delivers considerable welfare gains if we allow for compensation. Following the concept of Pareto efficiency in Brunnermeier et al. (2014), we can then say that a moderate constraint of re-use is Pareto improving compared to both extremes—the situation where re-use is prohibited and the situation where re-use is not regulated.

Showing that the moderate regulation of re-use constitutes the welfare optimum in a calibrated dynamic stochastic general equilibrium model constitutes an important contribution to the literature on re-use of collateral. In our analysis above we nevertheless focus on the qualitative features of the analysis. The quantitative results should be interpreted with caution as there are important pros and cons of regulation that are absent from the model. For instance, leaving re-use unregulated, thus allowing for high asset-price volatility, might impair financing conditions for firms and thereby reduce output and welfare. But then, regulating re-use might be costly or ineffective, while it can be perfectly enforced without cost in the model.

7 Conclusion

Publicly available data on the collateral re-use activity of globally active banks indicates that collateral re-use plays an important role in financial markets. While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. The use of the same security as underlying collateral for different transactions increases the sum of exposures in the financial system and, as result, creates leverage across the intermediation chain.

Our analysis is particularly relevant in light of the growing importance of collateral in the financial system and the need to specify the rules governing the re-use of such collateral in the regulatory framework. A range of regulations already impose direct limits and restrictions on the ability of financial intermediaries to reuse collateral. Moreover, rules affecting collateral transactions like repos may also indirectly limit the extent to which entities may reuse collateral. For example, the FSB (2017b) highlights that the Basel III reforms and in particular the leverage ratio (LR) framework are an important brake with which to address risk related to the contribution of the re-use of collateral to the buildup of leverage in the banking sector by indirectly incentivizing banks to keep re-use activity below excessive levels. To provide information for the policy discussion on whether collateral re-use should be permitted on financial markets—and if so, to what degree—it is clearly important to develop a model framework to understand the (qualitative) implications of collateral re-use on financial market outcomes and welfare.

In this paper, we have developed a calibrated infinite-horizon asset-pricing model with heterogeneous agents that allows us to assess the qualitative implications of re-use on financial market leverage, volatility, and welfare. In our model, the ability of agents to reuse frees up collateral that can be used to back more transactions. Through this channel, re-use of collateral contributes to the buildup of leverage in the financial system and is found to significantly increase volatility in financial markets. We have shown that limits on the amount of collateral that agents may reuse reduce financial market volatility; in fact, the tighter the limits, the lower the volatility. While the effect of re-use limits on volatility is monotone, the impact on welfare is not. In the model, allowing for some re-use can improve welfare as it enables agents to more effectively share risks. Allowing re-use beyond intermediate levels, however, can lead to excessive leverage and lower welfare. In conclusion, the analysis in this paper provides a rationale for limiting, yet not banning, re-use in financial markets.

Appendix

A Calibration with Large IES

While in the micro-consumption literature most studies tend to find an IES far below 1, and such values are used in most of the macroeconomics literature, an IES above 1 is often assumed in (macro-) finance papers. As a robustness check of our welfare results we therefore examine how our results depend on our choice of an IES of 0.5. For this purpose, we assume that both agents have an IES of 1.5. For this we are not able to match the second moment of equity returns observed in the data. Instead of fully re-calibrating the model we therefore only adjust the beta to 0.91 in order to approximately match the risk-free rate in the data for the case of unlimited re-use.

Table VII shows first and second moments of returns for our choice of parameters. We see that the equity premium is much higher in this calibration but that the standard deviation of equity returns is significantly lower, which is caused by a lower standard deviation of agent 1's financial wealth holdings. Nevertheless it seems fair to say that our parameterization produces realistic moments of asset returns. In particular it is still the case that the possibility of re-use has very large effects on

Table VII: Simulation statistics for the model with an IES of 1.5

	No re-use ($\kappa = 0$)	$\kappa = 0.2$	Free re-use ($\kappa = 1$)
Mean wealth, agent 1 (in %)	41	58	79
STD wealth, agent 1 (in %)	7.7	12	18
Mean re-use rate (in %)	0.0	18	33
Mean bond holding, agent 1	-0.9	-1.0	-1.2
Mean equity return (in %)	8.5	7.9	7.8
Mean risk-free rate (in %)	5.7	3.1	0.4
STD risky returns (in %)	7.1	8.6	13
STD risk-free returns (in %)	1.0	2.1	2.9

The table reports simulation statistics for our economic model with an IES of 1.5 for three different values of the re-use parameter κ. The abbreviation "STD" stands for "standard deviation".

stock-price volatility. Our first main meme of this paper, the idea that re-use has large effects on leverage and volatility, certainly remains correct for this alternative calibration.

Figure XI shows the analogue of Figure VII for this calibration with a large IES for both agents. The main advantage of this calibration is that the path for asset prices looks more "natural". In the calibration with a small IES a large part of the stock volatility was driven by booms and spikes in asset prices. With a large IES the volatility is driven partly by booms but mainly by busts—bad shocks cause asset prices to drop substantially.

We now turn to the welfare effects of regulating re-use. As is to be expected, the effects are very similar to the ones in our benchmark calibration. In fact, for this calibration there is a fairly substantial region of re-use (around $\kappa = 0.2$) for which regulation is Pareto improving (under our criterium) even without transfers.

Figure XII is the analogue of the right graph of Figure IX above for the case of an IES greater than 1. We again find an interior maximum welfare point at the 22.5 percent re-use limit. The explanation for the effect is identical to the explanation above. Shutting down differences in risk aversion means no re-use is always optimal while shutting down differences in beliefs means that full re-use is always optimal.

Finally as in Section 6.2 above we conduct the "full" welfare analysis and show that, as above, following the concept of Pareto efficiency in Brunnermeier et al. (2014), we can conclude that a moderate constraint of re-use is Pareto improving. Table VIII reports the welfare effects of regulation for five different values of beliefs. As above we verified the result for many other values in the convex hull.

Figure XI: Simulated paths of equilibrium quantities

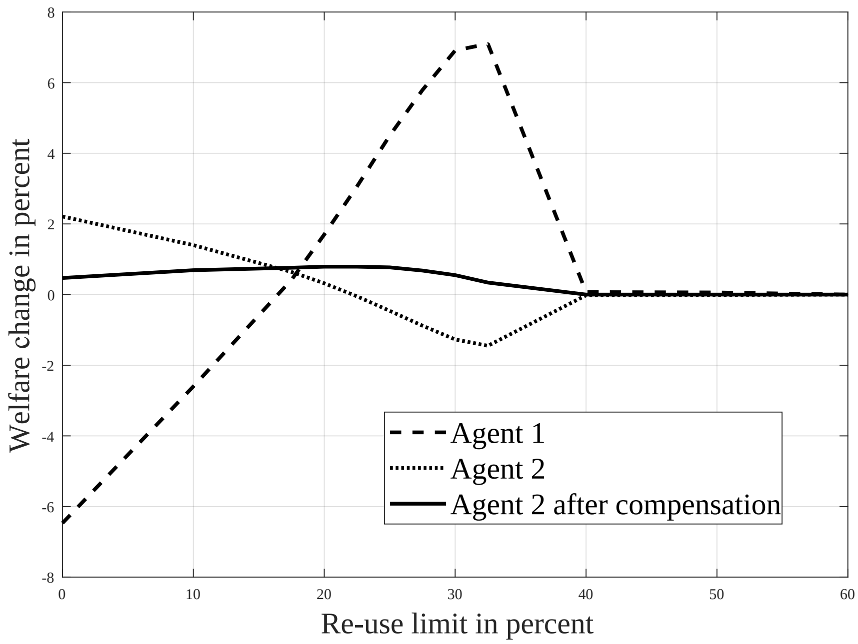
The figure shows simulated paths of six different equilibrium quantities over 100 periods. Disaster shocks occurred in periods 22, 33, 69, and 84. The graph on re-use of collateral shows the path of agent 2's short position in the stock, which is identical to $\max(0, \theta^1(s^t) - 1)$.

Table VIII: Sensitivity to choice of the beliefs used to evaluate welfare

Agent 1	Average 1	Benchmark	Average 2	Agent 2
32.5 (0.28)	27.5 (0.50)	22.5 (0.80)	12.5 (1.12)	12.5 (1.29)

The table reports the location of the peak of the welfare function in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at this location (in parentheses) for our economic model for the agents' value functions under different beliefs. The entry under "Benchmark" denotes the welfare change under true beliefs, the entry under "Average 1" denotes the welfare change for beliefs that are the average of agent 1's beliefs and the true beliefs, and so on.

Figure XII: Welfare changes as a function of the re-use parameter



The figure shows the welfare changes for both agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. The benchmark economy for the comparison is an unregulated economy in a time period when agent 1's wealth is equal to the mean of the ergodic distribution; also, the economy enters state 3 in the subsequent period.

B Simulations without Disasters

While it is a central feature of our calibrated model that the two types of agents have differing believes about the probability of disasters, it is not essential for our calibration exercise that these disasters actually occur. Table IX reports statistics for simulations in which no disasters occur. It turns out that they differ only surprisingly little from the statistics for the case with disasters in simulations (reported in Table IV). As expected, the mean equity return and the mean wealth of agent 1 are somewhat higher when there are no disasters that reduce them. When it comes to second moments, they are considerably lower for the cases with little or no re-use. However, in the case of free re-use the second moments in both tables are almost the same. This finding shows that the success of our calibration in matching the high volatility of equity returns does not rely on disasters occurring in the simulations.

Table IX: Simulation statistics for the model without re-use and with re-use when no disaster occurs

	no re-use ($\kappa = 0$)	some re-use ($\kappa = 0.2$)	free re-use ($\kappa = 1$)
mean wealth, agent 1 (in %)	36	51	68
STD wealth, agent 1 (in %)	3.0	4.3	16
mean re-use rate (in %)	0.0	20	34
mean bond holding, agent 1	-2.0	-2.3	-2.3
mean equity return (in %)	5.5	5.3	6.4
mean risk-free rate (in %)	2.7	1.9	0.4
STD equity returns (in %)	2.7	3.3	19
STD risk-free returns (in %)	0.2	0.1	3.6

The table reports statistics of simulations without disasters occurring for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$.

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