Amplification of aggregate shocks when entrepreneurs face collateral constraints and idiosyncratic risk

Johannes Brumm*
DBF, University of Zurich

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Abstract

This paper presents a calibrated general equilibrium model in which collateral constraints substantially amplify and propagate shocks to aggregate productivity. Compared to previous quantitative studies, the impact of collateral constraints is stronger and more robust because of their interaction with idiosyncratic risk. Agents' productivity as workers and entrepreneurs evolves stochastically, thereby creating a mismatch between accumulated wealth and current skills. As a consequence, collateral constraints bind for low-wealth entrepreneurs with high marginal returns, resulting in an inefficient allocation of capital. When negative shocks hit the economy, the allocation of capital deteriorates further, because the wealth of the constrained agents falls and their number rises. These dynamics amplify shocks substantially and imply that recessions are much sharper than booms.

Keywords: Amplification, Collateral Constraints, Entrepreneurship, Financial Frictions, Heterogeneity.

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1 Introduction

This paper presents a calibrated general equilibrium model in which collateral constraints on borrowing strongly amplify and propagate shocks to aggregate productivity. They do so through their impact on asset prices and capital allocation. To understand how, suppose a shock reduces productivity and hence the price of productive assets. Such a shock hits most those agents who borrowed heavily using their assets as collateral. If they already hold as much debt as the value of their collateral allows, they are forced to reduce their debt and sell assets. The resulting reallocation of capital is inefficient, thus diminishing aggregate output and reducing asset prices further. All in all, this constitutes a vicious circle depressing the economy—a mechanism that can explain why recessions are sharper than booms (see, e.g., Hamilton 1989), and also why they are larger and more persistent than the shocks hitting the economy (see, e.g., Cochrane 1994).

Kiyotaki and Moore (1997) present this mechanism in a very stylized model with two types of risk-neutral agents that differ in discount factors. The impatient agents face constant returns to scale and borrow from the patient ones as much as the collateral constraint allows. Subsequent papers relax the assumptions of linear preferences and technologies to be better able to quantify the impact of collateral constraints. Cordoba and Ripoll (2004b) and other studies document that amplification is generally small, being sizable only for extreme parameter values. Strikingly, most of these papers assume heterogeneity in patience to induce collateralized borrowing.

In this paper, agents do not differ with respect to preferences. Heterogeneity enters through idiosyncratic risk concerning their productivity as workers and entrepreneurs.[1] Consequently, borrowing occurs because idiosyncratic shocks create a perpetual mismatch between wealth and the entrepreneurial skills that are needed to make productive use of that wealth. In particular, the unproductive among the rich lend money to the productive among the poor.

Replacing heterogeneous patience by idiosyncratic risk has three major advantages when addressing the question raised by Kiyotaki and Moore (1997),

[1] Other papers that analyze aggregate fluctuations in a model with idiosyncratic risk and collateral constraints are Khan and Thomas (2013), Shourideh and Zetlin-Jones (2012), Liu et al. (2013), and Mendoza (2010). While the first three find only small effects of aggregate productivity shocks and thus focus on other types of shocks, Mendoza (2010) finds stronger effects in a model of a developing, small open economy.

namely how much collateral constraints amplify and propagate shocks to aggregate productivity. The first advantage is that amplification and propagation turn out to be stronger and more robust in the model with idiosyncratic risk than in models in which two types of agents differ in discount factors. The main reason for this surprising result is that in the latter models all borrowers are constrained. Therefore, a negative shock that reduces asset prices and thus the borrowing capacity of these agents implies a dramatic drop in the aggregate demand for credit. This causes a sharp decline in the interest rate, which mitigates the impact of collateral constraints, as Cordoba and Ripoll (2004b) point out. In contrast, the interest rate reacts only moderately in my model, as there are unconstrained borrowers who take up additional credit. Such moderate movements of the real interest rate are in line with its rather low volatility in the data (see, e.g., King and Watson 1996).

Second, idiosyncratic risk creates a distribution of wealth that is continuous rather than concentrated at two points. As a consequence, the percentage of constrained agents changes as shocks hit the economy. Such dynamics imply that a negative shock depresses the economy more than a positive shock of the same magnitude boosts it. This may help explain the empirical finding that recessions are sharper than booms (see, e.g., Hamilton 1989, Acemoglu and Scott 1997, and Hansen and Prescott 2005).

Finally, the process driving the evolution of skills can be empirically measured, whereas extreme differences in patience are not intended as a serious microfoundation for endogenously binding collateral constraints, as Kiyotaki and Moore (1997) admit: "A weakness of our model is that it provides no analysis of who becomes credit constrained, and when."

The details of the model are as follows: There is a continuum of infinitely lived agents with homogeneous preferences and heterogeneous skills. Their skills follow Markov processes that determine which agents are currently entrepreneurs and which are workers, either low skilled or high skilled. Workers save out of a precautionary motive, whereas entrepreneurs take up debt to buy capital. They employ their capital in agent-specific, linear technologies to create differentiated intermediate goods. These are then combined with labor to produce the final output good. As the intermediate goods are imperfect substitutes for each other, the price of a good falls as its supply rises, and thus entrepreneurs face decreasing returns to their investment in capital. Consequently, entrepreneurs with little wealth want to exploit their high marginal returns and thus borrow

as much as they are allowed to. The final good is used for consumption and investment in capital. Aggregate investment is assumed to be subject to convex adjustment costs; yet, to stay in line with most of the previous literature, the baseline calibration has a fixed capital stock. Agents trade capital and also risk-free debt. However, they may only borrow when holding enough capital as collateral. More precisely, the borrowing limit that each agent faces is proportionate to the current value of his capital holdings.
To analyze the propagation effect of collateral constraints, I follow much of the previous literature and consider an unanticipated shock that reduces total factor productivity (TFP) for a single period. Before the shock hits, the economy is in a (stochastic) steady state, where individual variables change, but aggregates are constant. After the shock, the economy moves away from the steady state and subsequently converges back to it. In the scenario in which TFP falls by 1 percent for 1 period, output and the price of capital are reduced strongly and persistently. One year after the shock, the price of capital is down by almost 5 percent and output by almost 1 percent. Four years after the shock, the price of capital is still depressed by 1 percent, and it takes 6 years for the economy to return, roughly, to its steady state. In contrast, with complete markets the economy would be back at the steady state within one year. Thus, collateral constraints propagate the shock substantially over time.
The mechanism behind this effect is the following: When the shock hits, the wealth of constrained agents falls by more than 10 percent, which makes them reduce their capital holdings by the same magnitude. Consequently, capital is allocated less efficiently, which drags down output directly and also indirectly through its impact on labor supply. In contrast to the previous literature, the dynamics are also driven by a change in the percentage of constrained agents, which goes up by more than 10 percent and takes several years to return to its steady state level. These endogenous dynamics of the model imply that the relation between the size of a shock and its impact is non-linear. For instance, if the size of a negative shock is doubled from -0.5 to -1.0 percent, then more agents are strongly affected by the shock, which in addition is twice as severe. Consequently, the impact on the aggregate economy is more than twice as large.

There is an extensive literature that incorporates financial frictions in macroeconomic models (see Quadrini 2011 and Brunnermeier et al. 2013 for excellent overviews). In the following, I focus on those strands of this literature that are most closely related to the present paper.

The mechanism by which collateral constraints propagate shocks through their impact on asset prices is theoretically analyzed by Kiyotaki and Moore (1997) and many others. Kocherlakota (2000) and Cordoba and Ripoll (2004b) quantify this mechanism and find that it has sizable effects only for extreme parameter values—a very high capital share combined with a very low elasticity of inter-temporal substitution. Pintus (2011), Mendicino (2012), and Punzi and Rabitsch (2015) generate more robust effects by assuming either heterogeneity in risk-aversion, persistent shocks and costly debt enforcement, or persistent shocks and heterogeneity in investors ability to borrow from collateral, respectively. Other studies extend Kiyotaki and Moore (1997) by replacing or complementing TFP shocks with other types of shocks. Cordoba and Ripoll (2004a) analyze monetary shocks, whereas Liu et al. (2013) analyze shocks to the demand for land, which is the collateral asset in their model. All these papers find that the respective shocks cause persistent movements in output. Combined with my results, these studies show that collateral constraints can have a sizable impact in a calibrated model if one deviates from Kiyotaki and Moore (1997) by considering different shocks *or* by replacing heterogeneous preferences with idiosyncratic risk. Shourideh and Zetlin-Jones (2012), Khan and Thomas (2013), and Buera et al. (2015) do both at the same time: they analyze the impact of aggregate *financial* shocks in models with heterogeneous firms facing idiosyncratic productivity risk.

Furthermore, the present paper is related to Moll (2014) and Buera and Moll (2015). They also model entrepreneurs who are subject to idiosyncratic productivity risk and collateral constraints. However, following Kiyotaki (1998) and Angeletos (2007), they assume that entrepreneurs face constant returns to scale. As a consequence, each individual entrepreneur is either constrained or does not produce at all, depending only on his productivity. While this is a drawback for quantitative analysis, it implies that individual decisions aggregate nicely, which makes strong theoretical results possible. One such result from Buera and Moll (2015) is that heterogeneous agent models with collateral constraints correspond to representative agent models featuring wedges, and that the type of wedge depends on the assumed heterogeneity: heterogeneous investment costs imply investment wedges, while heterogeneous productivity implies efficiency wedges. Chari et al. (2007) show that investment wedges do not account for a large part of the aggregate fluctuations found in US data, but efficiency wedges do. This result provides further support for the mechanism proposed in the present paper, where collateral constraints cause endogenous

movements in efficiency.
As the present paper, Bassetto et al. (2015) analyze the amplification of aggregate shocks in a model where agents face idiosyncratic risk concerning their skills as workers and entrepreneurs. Their model is in many dimensions richer than mine; for instance, it explicitly distinguishes between corporate and entrepreneurial firms and it has an endogenous choice of occupation. However, their credit constraint differs from the standard collateral constraint and their analysis focuses on shocks to financial intermediation.[2] In these respects, the present paper is deliberately kept closer to the original setup of Kiyotaki and Moore (1997).
An aspect in which both the present paper and Bassetto et al. (2015) follow Kiyotaki and Moore (1997) is in considering unanticipated aggregate shocks rather than anticipated ones. The standard method for approximating recursive equilibrium in models with anticipated aggregate shocks by Krusell and Smith (1998) assumes bounded rationality to reduce the state space, approximating the wealth distribution by the first moment of aggregate capital. However, in my model next period's factor prices—which agents need to forecast to make optimal choices—are not simply a function of the aggregate capital stock, but also crucially depend on the allocation of capital. Therefore, using only aggregate capital is unlikely to provide a sufficiently good approximation to the true equilibrium.[3] For this reason, and also to stay close to the original question posed by Kiyotaki and Moore (1997), the present paper rather approximates the full rational expectations equilibrium of the model with unanticipated shocks—which is a substantial computational challenge, as three aggregate variables change along the transition path, instead of only one when the transitional dynamics of an Aiyagari model is computed (see Appendix A.2).

The remainder of this paper is organized as follows: Section 2 presents the model and analytic results. Section 3 describes the calibration of the model. Section 4 studies the steady state, analyzes the response to (non-persistent and persistent) TFP shocks, carries out a sensitivity analysis, and compares the results to those of previous papers. Section 5 concludes.

[2] Their credit constraint is derived from a limited commitment assumption and includes profits—it is thus not linear in the value of collateral as in Kiyotaki and Moore (1997). In addition to a careful analysis of the effect of financial shocks, they also briefly consider a large shock to TFP (of −2.5%).

[3] Alleviating this problem by including higher moments of the wealth distribution in the state space is beyond the scope of this paper, yet could be achieved with methods from Reiter (2009), Brumm and Grill (2014), or Brumm and Scheidegger (2015).

2 Model

Time is discrete and infinite. The economy is populated by a continuum of infinitely lived agents with homogeneous preferences and heterogeneous skills. While there is no aggregate risk, skills are subject to idiosyncratic risk. Depending on their skills, some agents are workers while others are entrepreneurs. The latter use capital to produce intermediate goods, which are then combined with labor to produce the final output good. This good is used for consumption as well as investment in capital, which is subject to convex adjustment costs. Financial markets are incomplete: Agents trade in capital and debt subject to collateral constraints. The details follow.

2.1 Preferences and Skills

Agents have homogeneous preferences: The discount factor, β, the risk aversion parameter, γ, and the elasticity of labor supply, $1/\theta$, are all equal across agents. Each agent $i \in [0, 1]$ maximizes expected lifetime utility given by

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, h_{i,t}\right)\right],$$

where $c_{i,t}$ and $h_{i,t}$ are consumption and hours worked of agent i at time t, respectively. Per-period utility is assumed to be of the Greenwood–Hercowitz–Huffmann (GHH) type:

$$u\left(c_{i,t}, h_{i,t}\right) = \frac{1}{1-\gamma}\left(c_{i,t} - \frac{{h_{i,t}}^{1+\theta}}{1+\theta}\right)^{1-\gamma}.$$

This specification excludes any wealth effect on the choice of hours worked (see Greenwood et al. (1988)). As a consequence, aggregate labor supply does not depend on the distribution of wealth among agents (see Result 2).

Agents differ with respect to their labor productivity, $a^w_{i,t} \in A^w$, and their entrepreneurial productivity, $a^e_{i,t} \in A^e$. Agent's types, $s_{i,t} = \left(a^w_{i,t}, a^e_{i,t}\right) \in S$, follow Markov processes that are independent and identically distributed across agents. Their transition probabilities are denoted by $M(s_{i,t}, s_{i,t+1})$. The distribution over types is assumed to be stationary, and the respective measure over types is denoted by μ. At time t agent i supplies $a^w_{i,t} h_{i,t}$ units of labor to the market. If $a^e_{i,t} > 0$, then agent i also runs a business as explained below. Therefore, this agent is referred to as an entrepreneur. All other agents are referred to as workers.

2.2 Production

Each entrepreneur may invest to produce a differentiated intermediate good. The amount produced,

$$y_{i,t} = f(a^e_{i,t-1}, a^e_{i,t})k_{i,t},$$

is linear in the capital invested, $k_{i,t}$, which is chosen in period $t-1$. Through the function f, production depends on the entrepreneurial productivity in the period of investment and the period of production. I assume that $f(0, \cdot) = 0$, thus workers do not have an investment opportunity.

Final output, Y_t, is produced competitively from labor,[4]

$$L_t = \int a^w_{i,t} h_{i,t} di,$$

and intermediate goods, $y_{i,t}$, where total factor productivity is A_t:

$$Y_t = A_t \left(\int y^{\phi}_{i,t} di \right)^{\alpha/\phi} L_t^{1-\alpha}.$$

The elasticity of substitution between intermediate goods in final-good production is $1/(1-\phi)$. I assume $0 < \phi < 1$, thus intermediate goods are imperfect substitutes. Final output is used for consumption and investment:

$$Y_t = C_t + I_t.$$

Apart from labor, capital is the only productive factor in this model. This is in contrast to Kiyotaki and Moore (1997) and Kocherlakota (2000) who make the stark distinction between non-depreciating land and fully depreciating capital. Omitting this distinction but assuming partial depreciation, my model is closer to standard quantitative macro models. However, for the collateral constraint mechanism to work, the price of collateral (that is of capital) has to fall in bad

[4]Labor is assumed to enter final-good production rather than intermediate-good production, because this ensures that the labor share in aggregate output does not depend on the distribution of capital among entrepreneurs. This property is needed to establish Result 2 in Proposition 1, which simplifies the numerical solution considerably. If labor entered intermediate-good production instead, this would not make a significant quantitative difference as long as labor does not have to be paid in advance by entrepreneurs. However, if labor had to be paid in advance, then collateral constraints would be tighter and the effects of collateral constraints would probably be even stronger, as the sensitivity analysis in Section 4.3 suggests.

times. The most standard assumption that generates this reaction is convex adjustment costs as in Hayashi (1982). In particular, I assume quadratic costs as in Lorenzoni and Walentin (2007). There is a competitive capital production sector that produces K_{t+1} units of new capital from combining K_t units of old capital with an amount of investment given by[5]

$$I_t(K_t, K_{t+1}) = K_{t+1} - (1-\delta)K_t + \frac{\xi}{2}\frac{(K_{t+1}-K_t)^2}{K_t},$$

where $\delta \in [0,1]$ is depreciation and $\xi \in [0,+\infty]$ parameterizes adjustment costs.[6] This specification includes two important special cases: for $\xi = 0$, standard neoclassical capital accumulation; for $\xi = +\infty$, a fixed capital stock.

2.3 Markets

The consumption good is traded at the normalized price of 1. The price of the intermediate good produced by agent i at time t is denoted by $\pi_{i,t}$. The price of old capital, k_t, sold to the capital production sector at t is p_t, the price charged for new capital, k_{t+1}, is q_t. Agents may take up debt, $d_{i,t+1}$, in which case they have to repay $d_{i,t+1}R_{t+1}$ at $t+1$; thus R_{t+1} is the gross interest rate from t to $t+1$. Consequently, the budget constraint (BC) faced by each agent is given by

$$\text{BC:} \qquad c_t + q_t k_{t+1} - d_{t+1} \leq \pi_t k_t f(a^e_{t-1}, a^e_t) + p_t k_t - d_t R_t + w_t h_t a^w_t.$$

[5]Alternatively, one could make the adjustment costs proportional to $(K_{t+1} - K_t(1-\delta))^2$ rather than $(K_{t+1} - K_t)^2$. While both specifications are intuitive, the latter is technically more convenient. First, it makes the steady state independent of the parameter ξ. Second, the extreme case of $\xi = +\infty$ corresponds to a fixed capital stock, rather than a capital stock that decays by the rate of depreciation.

[6]In Lorenzoni and Walentin (2007) each individual entrepreneur faces an adjustment costs function as specified above. The authors also assume that old capital may be traded among entrepreneurs before investments are made. With this assumption, all entrepreneurs choose the same ratio of old capital to new capital and thus face the same shadow cost of new capital. For this reason, the specification in Lorenzoni and Walentin (2007) is equivalent to my assumption—namely, a competitive capital production sector that buys all old capital and sells new capital. I choose to assume convex adjustment costs at the *aggregate* level, because empirical studies show that convex adjustment costs are consistent with aggregate investment data, but not with firm-level data (see Bloom et al. 2007). Wang and Wen (2012) provide a microfoundation for convex adjustment costs at the aggregate level.

Debt is in zero net supply,

$$D_{t+1} = \int d_{i,t+1} di = 0,$$

thus some agents hold negative debt thereby lending money to other agents. However, lenders cannot force borrowers to repay their debts unless these debts are secured by collateral assets. Therefore, they impose the following collateral constraint on borrowers:

$$\text{CC:} \qquad d_{t+1}R_{t+1} \leq \kappa p_t k_{t+1}, \text{ where } \kappa \in [0, 1].$$

In words, the repayment obligation of a borrower may not exceed a fraction κ of the depreciated value of the capital acquired today. In this constraint, lenders (or regulators) use the current price of old capital, p_t, to assess the collateral value and set the borrowing limit accordingly. While this is a common modeling choice (see, e.g., Mendoza 2010, Garin 2015, and Punzi and Rabitsch 2015) that seems to be in line with prevalent market practices (like contracts with a margin clause), it is less convincing from a theoretical perspective. Therefore, I also consider (in Section 4.3) the case in which next period's price of old capital, p_{t+1}, is used to assess the collateral value:

$$d_{t+1}R_{t+1} \leq \kappa p_{t+1} k_{t+1}.$$

The justification for considering values of κ below 1 is that liquidation of the collateralized assets might be inefficient. More precisely, in the case of default, lenders can recover only a fraction κ of the collateral value. Knowing this, they are not willing to lend more than this fraction in the first place.
Finally, agents face a short-sale constraint (SC) on capital:

$$\text{SC:} \qquad 0 \leq k_{t+1}.$$

2.4 Equilibrium

I now define equilibrium for the economy described above. In the below definition, agents' policies do not depend on i, but only on time t and individual characteristics, $x_t \equiv (d_t, k_t, a_t^w, a_t^e, a_{t-1}^e)$. Consequently, idiosyncratic quantities ($c_t, d_{t+1}, h_t, k_{t+1}, y_t$, and π_t) in the below definition are functions that map the state of an agent, x_t, to a real number. In contrast, prices (p_t, q_t, R_{t+1}, and w_t) and aggregate quantities (K_t, L_t, Y_t) are just real numbers. The distribution over individual characteristics is denoted by Φ_t. Recall that there is no aggregate risk, thus expectations are over idiosyncratic shocks only.

Definition 1 (Competitive Equilibrium)
A competitive equilibrium of the economy $\langle \beta, \gamma, \theta, S, M, f, \{A_t\}, \alpha, \phi, \delta, \xi, \kappa, \Phi_0 \rangle$ *is a sequence of quantities, prices, and distributions*[7]

$$\{c_t, d_{t+1}, h_t, k_{t+1}, y_t, K_t, L_t, Y_t, p_t, q_t, R_{t+1}, w_t, \pi_t, \Phi_t\}_{t\in\mathbb{N}}$$

such that:

1. *Given prices* $\{p_t, q_t, R_{t+1}, w_t, \pi_t\}_{t\in\mathbb{N}}$*, agents choose quantities* $\{c_t, d_{t+1}, h_t, k_{t+1}\}_{t\in\mathbb{N}}$ *to maximize*

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}\left(c_t - \frac{{h_t}^{1+\theta}}{1+\theta}\right)^{1-\gamma}\right],$$

   subject to BC, CC, and SC.

2. *For all* $t \in \mathbb{N}$*, given prices* (π_t, w_t)*, the final output firm chooses inputs* (y_t, L_t) *to maximize profits*

$$A_t \left(\int y_t(x)^{\phi} d\Phi_t(x)\right)^{\alpha/\phi} L_t^{1-\alpha} - \int \pi_t(x) y_t(x) d\Phi_t(x) - w_t L_t.$$

3. *For all* $t \in \mathbb{N}$*, given prices* (q_t, p_t)*, the capital production firm chooses inputs* (I_t, K_t) *to maximize profits*

$$q_t K_{t+1}(I_t, K_t) - I_t - p_t K_t,$$

   where $K_{t+1}(I_t, K_t)$ *is given by* $I_t = K_{t+1} - (1-\delta)K_t + \frac{\xi}{2}\frac{(K_{t+1}-K_t)^2}{K_t}$*.*

4. *For all* $t \in \mathbb{N}$*, all markets clear:*[8]

$$\begin{aligned}
&L_t = \int a_t^w(x) h_t(x) d\Phi_t(x), \;\; Y_t = C_t + I_t, \\
&K_t = \int k_t(x) d\Phi_t(x), \;\; K_{t+1} = \int k_{t+1}(x) d\Phi_t(x), \;\; 0 = \int d_{t+1}(x) d\Phi_t(x), \\
&\forall s: \quad y_t(x) = f(a_{t-1}^e(x), a_t^e(x)) k_t(x).
\end{aligned}$$

[7] To streamline the exposition, I omit requirements for the measurability of the policy functions. Note also that I use the definition $\mathbb{N} = \{0, 1, \ldots\}$.

[8] Note that K_{t+1} denotes new capital in period t and also old capital in period $t+1$. Hence, it has to satisfy the two respective market clearing conditions. Note also that I use Walras's Law to omit the market clearing condition for the consumption good.

5. *For all* $t \in \mathbb{N}$, Φ_{t+1} *is generated from* Φ_t *by the exogenous Markov processes for skills and from individual policies* d_{t+1}, k_{t+1}.

For the computational exercises presented below, the natural starting point is a stationary equilibrium, which I also call a (stochastic) steady state. It is defined as follows:

Definition 2 (Stationary Competitive Equilibrium)
A stationary competitive equilibrium of the economy $\langle \beta, \gamma, \theta, S, M, f, A, \alpha, \phi, \delta, \xi, \kappa, \Phi_0 \rangle$ *is a competitive equilibrium*

$$\{c_t, d_{t+1}, h_t, k_{t+1}, y_t, K_t, L_t, Y_t, p_t, q_t, R_{t+1}, w_t, \pi_t, \Phi_t\}_{t\in\mathbb{N}}$$

with all components being constant over time.

To understand this definition, recall that among the equilibrium objects c_t, d_{t+1}, h_t, k_{t+1}, y_t, and π_t are functions that map the state of an agent, x_t, to a real number. In contrast, p_t, q_t, R_{t+1}, K_t, L_t, and Y_t are just real numbers.
To generate the results in Section 4, I have to numerically solve for competitive equilibria. Nevertheless, I first establish some analytical results, which simplify computations and provide intuition. These results are stated in Proposition 1, while their proofs are provided in Appendix A.1.

Proposition 1 (List of Analytical Results)
In a competitive equilibrium

$$\{c_t, d_{t+1}, h_t, k_{t+1}, y_t, K_t, L_t, Y_t, p_t, q_t, R_{t+1}, w_t, \pi_t, \Phi_t\}_{t\in\mathbb{N}}$$

of the economy $\langle \beta, \gamma, \theta, S, M, f, \{A_t\}, \alpha, \phi, \delta, \xi, \kappa, \Phi_0 \rangle$ *the following properties hold:*

- ***Result 1 (FOCs of the Individual Problem)***
  The FOCs of the individual optimization problem are for all $t \in \mathbb{N}$*:*

$$\begin{aligned}
BC:&\quad 0 = \pi_t k_t f(a^e_{t-1}, a^e_t) + p_t k_t - d_t R_t + w_t h_t a^w_t - c_t - q_t k_{t+1} + d_{t+1}\\
CC:&\quad 0 \le \kappa p_t k_{t+1} - d_{t+1} R_{t+t} \ \wedge\ 0 \le \lambda_t \ \wedge\ 0 = (\kappa p_t k_{t+1} - d_{t+1} R_{t+1})\,\lambda_t\\
SC:&\quad 0 \le k_{t+1} \ \wedge\ 0 \le \nu_t \ \wedge\ 0 = k_{t+1}\nu_t\\
k_{t+1}:&\quad 0 = -u_c(c_t, h_t) q_t + \nu_t + \lambda_t \kappa p_t\\
&\qquad + \beta \mathbb{E}\left[u_c(c_{t+1}, h_{t+1}) \left(\frac{\partial\,(\pi_{t+1} k_{t+1})}{\partial k_{t+1}} f(a^e_t, a^e_{t+1}) + p_{t+1}\right)\right]\\
d_{t+1}:&\quad 0 = u_c(c_t, h_t) - \lambda_t R_{t+1} - \beta R_{t+1} \mathbb{E}\left[u_c(c_{t+1}, h_{t+1})\right]\\
h_t:&\quad 0 = u_h(c_t, h_t) + u_c(c_t, h_t) w a^w_t
\end{aligned}$$

- ***Result 2 (Labor Supply and Wage)***
  $L_t = ((1-\alpha)\, Y_t)^{\frac{1}{1+\theta}}$, $w_t = ((1-\alpha)\, Y_t)^{\frac{\theta}{1+\theta}}$.

- ***Result 3 (Price of Intermediate Goods)***
  The price of intermediate goods $y(x_t)$ is given by
  $\pi(x_t) = Zy_t(x_t)^{\phi-1}$, *where* $Z \equiv \alpha\,(1-\alpha)^{\frac{(1-\alpha)\phi}{\alpha(1+\theta)}}\, A^{\phi/\alpha} Y^{\frac{\alpha-\phi}{\alpha}+\frac{(1-\alpha)\phi}{\alpha(1+\theta)}}$.

- ***Result 4 (Price of Capital)***
  The prices of old and new capital are given by

$$p_t = (1-\delta) + \frac{\xi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2 - 1\right), \quad q_t = 1 + \xi\left(\frac{K_{t+1}}{K_t} - 1\right).$$

- ***Result 5 (Price of Fixed Capital)***
  As adjustment costs ξ go to infinity, the prices of old and new capital satisfy: $p_t = q_t - \delta$.

Result 1 just reports the FOCs of the individual problem. For Result 2, I exploit the GHH preference specification to derive labor supply and the wage as functions of aggregate output only. In the proof, I make use of the following normalization concerning the distribution of labor productivity:

$$\int (a_{i,t}^w)^{1+1/\theta} di = 1.$$

This normalization ensures that aggregate labor supply is the same as in an economy in which all agents have a homogeneous labor productivity of 1. Result 3 gives the prices of intermediate goods. These are decreasing functions of the capital employed, implying that entrepreneurs face decreasing returns to investment. Result 4 shows that the prices of old and new capital are both increasing functions of the growth in aggregate capital, which implies that prices fall in bad times. Finally, Result 5 is concerned with the case of a fixed capital stock, which is the baseline case considered in Section 4. To make this assumption a special case of the economy with convex adjustment costs, the limit for ξ going to infinity, has to be considered. Being slightly imprecise, I refer to this limit as $\xi = +\infty$. According to Result 5, the prices of old and new capital differ exactly by δ if $\xi = +\infty$.

3 Calibration

I calibrate the model to annual data.[9] The parameters of the model, which are reported in Table 1, fall into four classes, relating to preferences, skills, production, and capital markets. Concerning preferences, I pick parameter values that are standard in the literature. The discount factor, β, is set to 0.931, which matches a real interest rate of 2.0%.[10] As debt is risk-free in this model, the model interest rate should be close to the actual risk-free rate. On the other hand, debt is the only investment opportunity for non-entrepreneurs in this economy, which speaks in favor of setting it equal to the (weighted) average of the return on risk-free and risky investments. To strike a compromise between these two lines of reasoning, I choose 2.0%. The parameter γ is set equal to 2, which is a moderate level of risk aversion. Finally, θ is taken to be 1/1.9, which implies a Frisch elasticity of 1.9. Following Hall (2011) and Hall (2009) this value of the Frisch elasticity is meant to capture both the lower elasticity of hours worked by the employed and the elasticity of employment resulting from sticky compensation (in a search-and-matching model following Mortensen and Pissarides 1994). In this way, the model generates a realistic co-movement of labor and output without explicitly assuming frictions in the labor market.[11] Our chosen value is at the lower end of macro estimates reported in Chetty et al. (2011). It turns out that elastic labor supply reinforces propagation generated by credit constraints (see Sections 4.2 and 4.3), while it does not generate propagation on its own (see Appendix A.3).

With regard to skills, I assume that the Markov process for labor productivity and entrepreneurial productivity, $s = (a^w, a^e)$, has a support of three states only:

$$s^l = (a^w_l, 0), \quad s^h = (a^w_h, 0), \quad s^e = (a^w_h, a^e_h).$$

[9] One important reason for choosing the period length to be 1 year rather than 3 months is as follows: The length of a period fixes the duration of debt contracts, which clearly plays a role in models with collateral constraints. A duration of 1 year is a much better approximation to the actual maturity structure of corporate debt than a duration of 1 quarter—Barclay and Smith (1995) report that more than 70% of outstanding corporate debt is due in more than 1 year.

[10] Note that the interest rate is markedly below $\beta^{-1} - 1 \approx 7.4\%$, which is the interest rate in the complete markets benchmark. The reason for this is that idiosyncratic risk induces substantial precautionary savings in this model.

[11] See Buera et al. (2015) or Garin (2015) for a model with frictions in both credit and labor markets.

preferences:			
discount factor	β	0.931	match 2% real interest rate
coefficient of risk aversion	γ	2	standard
Frisch elasticity	$1/\theta$	1.9	Hall (2009, 2011)
skills:			
productivity types	s^l s^h s^e	$(0.88, 0)$ $(1.08, 0)$ $(1.08, 2.63)$	see text
transition matrix	M	$\begin{pmatrix} .756 & .233 & .011 \\ .233 & .756 & .011 \\ .050 & .050 & .900 \end{pmatrix}$	see text
production:			
capital share	α	0.36	standard
elasticity of substitution betw. intermediate goods	$\frac{1}{1-\phi}$	4	in line with micro data, Burstein and Hellwig (2008)
depreciation	δ	0.05	match capital-output-ratio of 3
capital adjustment costs	ξ	$+\infty$	as in the previous literature
capital markets:			
collateralizability of capital	κ	0.76	Djankov et al. (2008)

Table 1: Baseline calibration

Agents with these skill levels are referred to as low-skilled workers, high-skilled workers, and entrepreneurs.[12] The two levels for labor productivity, a_l^w and a_h^w, are determined by a two-state approximation to the first-order autoregression of (log) individual labor income reported in Heaton and Lucas (1996), which has persistence $\rho = 0.529$.[13] While this value is at the lower end of what is used

[12] The simplifying assumption that there is only one (non-zero) level of entrepreneurial productivity is relaxed in Section 4.3. The assumption that entrepreneurs have high labor productivity is innocuous. Assuming low labor productivity for entrepreneurs tightens their collateral constraints slightly, yet hardly changes aggregate dynamics.

[13] Consider a discrete Markov chain for the log of labor income with states $\{-\epsilon, +\epsilon\}$ and transition probabilities $\mathbb{P}(\epsilon|\epsilon) = \mathbb{P}(-\epsilon| - \epsilon)$. I want this process to match moments of the autoregressive process from Heaton and Lucas (1996), which has persistence $\rho = 0.529$ and an error term with standard deviation $\sigma = 0.251$. In particular, the discrete Markov chain has to match the variance and the autocovariance: $\epsilon^2 = \sigma^2/(1-\rho^2)$, and $(2\mathbb{P}(\epsilon|\epsilon) - 1)\epsilon^2 = \rho\sigma^2/(1-\rho^2)$. This implies $\epsilon = \sqrt{\sigma^2/(1-\rho^2)}$ and $\mathbb{P}(\epsilon|\epsilon) = (1+\rho)/2$. The states $\{-\epsilon, +\epsilon\}$ are normalized values for the log of labor income (normalized such that the two states sum up to zero). To find the corresponding values for labor productivity, (a_h^w, a_l^w), I use that individual labor income is given by $wh_t a_t^w = (wa_t^w)^{1+1/\theta}$ (see proof of Result 2). Consequently, (a_h^w, a_l^w) have to satisfy: $(a_h^w)^{1+1/\theta}/(a_l^w)^{1+1/\theta} = e^{+\epsilon}/e^{-\epsilon}$. Combining this condition with the normalizing assumption about the distribution of a^w from Section 2, which now reads

in the literature, I show in Section 4.3 that the results of this paper are robust to assuming a much higher persistence. Concerning entrepreneurs, I assume that they make up 10% of the population and that the yearly exit rate from the state of being an entrepreneur is 10%. These choices are compromises between the respective values used by Quadrini (2000) and Bohácek (2006).[14] If an entrepreneur becomes a worker, the chances of being low skilled or high skilled are equal. Likewise, the probability of becoming an entrepreneur is the same for both types of workers. Combined, all these properties uniquely determine the transition matrix between individual states, M, which is given in Table 1. Concerning intermediate-good production, I assume

$$f(a^e_{i,t-1}, a^e_{i,t}) = a^e_{i,t} 1_{\{a^e_{i,t-1}>0\}} \text{ , i.e.} \quad y_{i,t} = \begin{cases} a^e_{i,t} k_{i,t} & \text{for} \quad a^e_{i,t-1} > 0 \\ 0 & \text{for} \quad a^e_{i,t-1} = 0 \text{ .} \end{cases}$$

Similar to Quadrini (2000), this entrepreneurial production function exhibits a high correlation between output and investment opportunities: $a^e_t = 0$ implies that current output is zero and that there are also no entrepreneurial investment opportunities. As a normalization, I set $a^e_h = M(s^e, s^e)^{-1/\phi} \mu(s^e)^{(\phi-1)/\phi}$. This ensures that, in the complete markets benchmark, aggregate output is as in an economy in which all agents are entrepreneurs with entrepreneurial productivity equal to 1, and the production function collapses to $Y_t = A_t K_t L_t$ (see Appendix A.3).

To match a capital-to-output ratio of 3.0, I set $A = 1$ and $\delta = 0.05$. The capital share, α, equals 0.36, as in Aiyagari (1994). The elasticity of substitution between intermediate goods is set equal to 4 (i.e., $\phi = .75$), which is well within the range of estimates from micro data (see Hsieh and Klenow 2009 and Burstein and Hellwig 2008).

Concerning adjustment costs, I make an extreme assumption for the baseline calibration—namely, $\xi = +\infty$. This results in a fixed capital stock, as assumed in, among others, Cordoba and Ripoll (2004b), Mendicino (2012), Punzi and Rabitsch (2015), and also the basic model of Kiyotaki and Moore (1997), where the fixed input is called land. Thus, choosing $\xi = +\infty$ makes my results comparable to many papers from the literature on collateral constraints. Additionally, from an empirical perspective, one could argue that capital is supplied almost inelastically in the short run. The reason for this is that it takes on average

$(a^w_h)^{1+1/\theta}/2 + (a^w_l)^{1+1/\theta}/2 = 1$, the parameters (a^w_h, a^w_l) are pinned down.

[14] In Quadrini (2000) the percentage of entrepreneurs is 12% and the yearly exit rate 18%. In Bohácek (2006) the percentage of entrepreneurs is 9% and the yearly exit rate 4.5%.

much longer than one year from the decision to invest until the completion of the investment (see Kydland and Prescott 1982 and Boca et al. 2008). Given that modeling this time-to-build lag is computationally burdensome, assuming infinite adjustment costs seems to be an acceptable shortcut for the question considered. In Section 4.3, I analyze the sensitivity of the results with respect to the adjustment cost parameter.

The only parameter of the model that relates to capital markets is κ. I follow Mendicino (2012) and base this parameter value on a measurement of the efficiency of debt enforcement from Djankov et al. (2008). In particular, I set $\kappa = 0.76$, which is the average measurement for OECD countries.

4 Results

This section presents a quantitative analysis of the model described above. The main exercise analyzes a one-time unanticipated shock to aggregate TFP, as in the previous literature on collateral constraints. The economy starts from a steady state—that is, a stationary competitive equilibrium as in Definition 2. Then TFP drops for 1 period, which is not anticipated. Starting from that period, the economy is on a transition path back to the steady state. Along the transition path the economy is in a non-stationary competitive equilibrium as in Definition 1. In this section, I first describe the steady state before I thoroughly study the impact generated by a shock. Then, I check how robust the results are to changes in crucial parameters and also how the dynamics changes when the shock is persistent. Finally, I discuss why there are stronger effects of collateral constraints in my model than found in previous studies. Appendix A.2 describes the numerical procedures used to compute the steady state and the transition path.

4.1 Steady State

While individual variables move in the steady state, their distribution does not vary. Figure 1 plots the distribution of financial wealth,

$$\omega_t \equiv \pi_t k_t f(a^e_{t-1}, a^e_t) + p_t k_t - d_t R_t,$$

for workers and entrepreneurs—normalized by the mean in the overall population. As high-skilled workers are just a little richer than low skilled ones, the

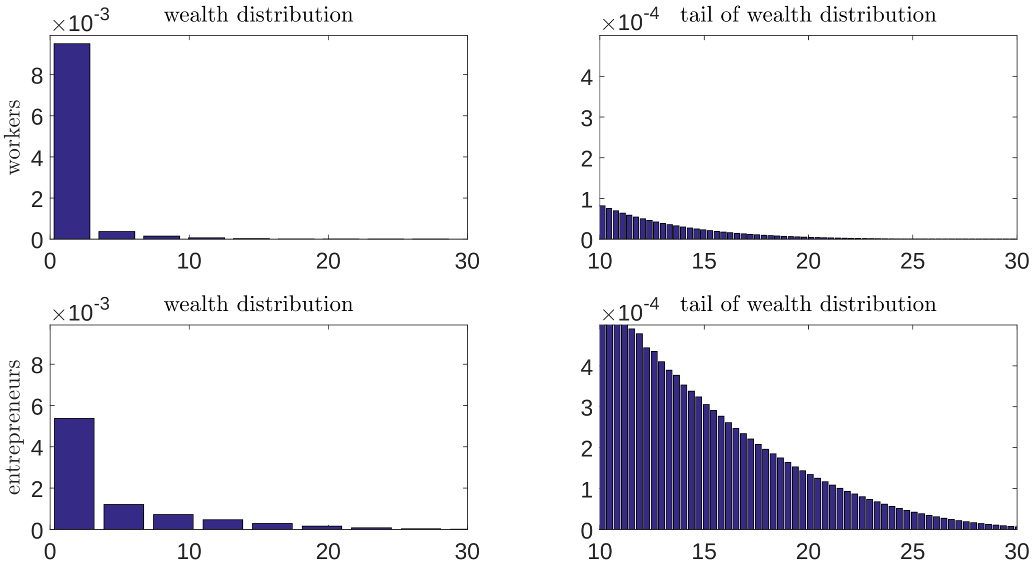

Figure 1: Distribution of wealth in steady state. Workers in the first row, entrepreneurs in the second row. The entire distribution is reported in column 1, while the second column shows the right tails. Wealth is reported in multiples of the mean in the overall population.

two types are not plotted separately. In contrast, the average entrepreneur has about six times more financial wealth than the average worker.

There are two important reasons for this large gap in wealth. First, entrepreneurs make business profits on top of labor earnings. Second, they have a stronger incentive to save, because they have high-return investment opportunities. To make use of these, they take up debt but also have to inject their own wealth as equity. This is displayed in Figure 2, which plots the policy functions of entrepreneurs. For levels of (normalized) financial wealth below about 6, which applies to 74% of all entrepreneurs, the collateral constraint is binding. In this situation, it follows from (CC) that debt is proportional to capital:

$$d_{t+1} = \frac{\kappa p_t}{R_{t+1}} k_{t+1}.$$

Thus, the fraction of an investment that agents can finance by borrowing is given by $\kappa p_t / R_{t+1} q_t$, which amounts to 71% in the baseline calibration. At the point where the collateral constraint ceases to be binding, there are kinks in the policy functions. From that point onward, investment in capital moderately increases further, while the demand for debt soon starts to decrease. The reason

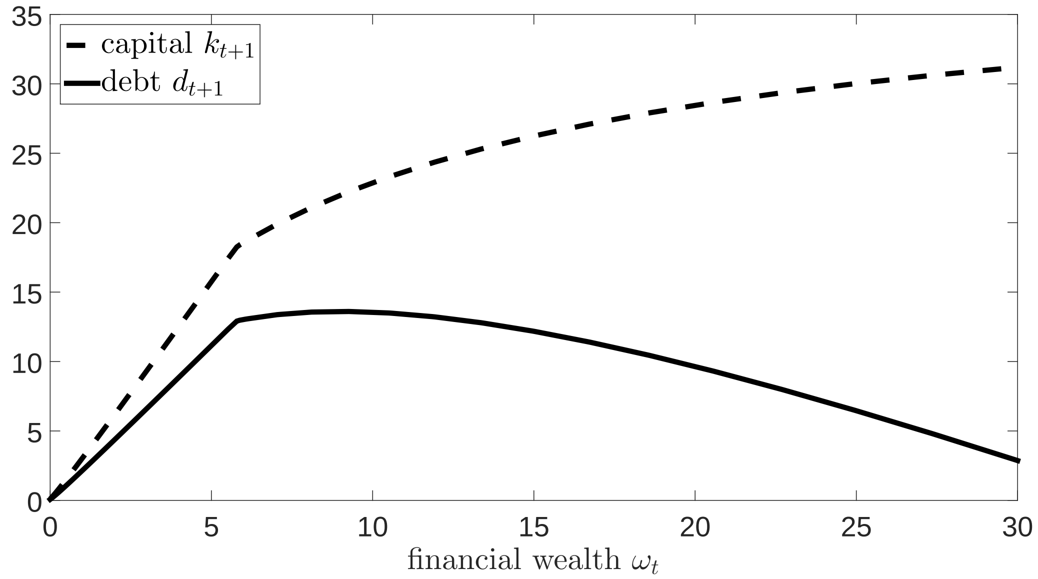

Figure 2: Policy functions of entrepreneurs in steady state.

for this is that very rich entrepreneurs finance a large part of their investments out of their own pockets.

While the calibration presented in Section 3 did not target the wealth distribution, it nevertheless implies statistics that are in line with data from the Survey of Consumer Finance (SCF). In the steady state of the model, entrepreneurs, who make up 10% of the population, hold 40% of total wealth, and the richest 5% of the population hold 51% of total wealth. In the SCF the self-employed, who make up 11% of the population, hold 39% of total wealth, and the richest 5% of the population hold 54% of total wealth (see Cagetti and De Nardi 2006).

4.2 Response to a (Non-Persistent) TFP Shock

I now analyze the response of the model economy to an unanticipated, non-persistent shock to aggregate TFP.[15] The shock happens in period $t = 0$. From $t = 1$ onward, TFP is assumed to be back at its steady state level. With such a shock, output would also return to its steady state level in period $t = 1$ already

[15] In this paper, A_t is called TFP as it linearly enters the aggregate production function. Thus, a temporary change in A_t is called a TFP shock. Note, however, that in this model A_t does not correspond to the empirically measured TFP, because of the endogenous changes in the allocation of capital that are discussed below.

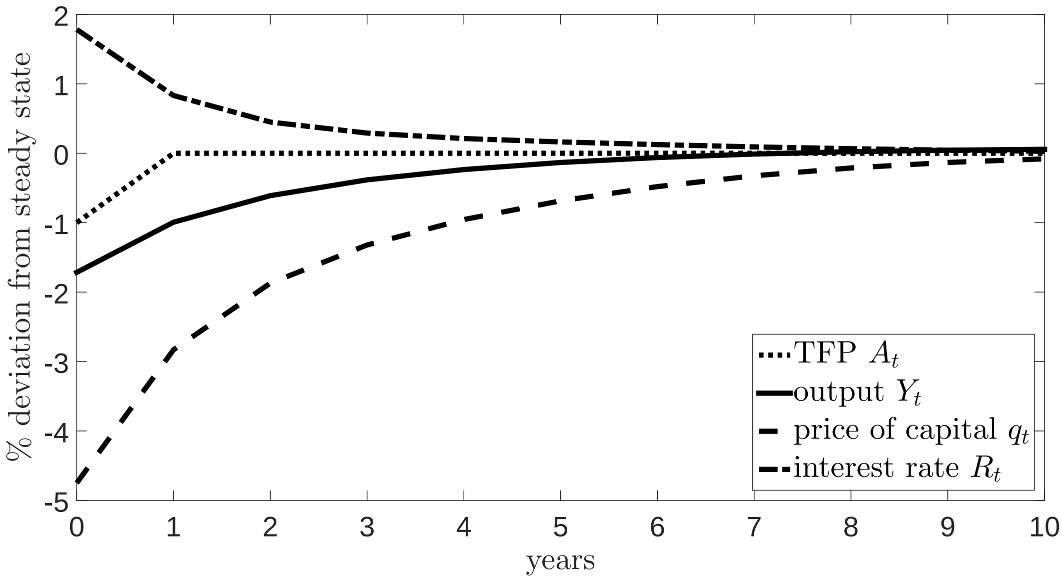

Figure 3: Response of output and prices to one-time -1 percent shock to TFP.

if markets were complete (see Appendix A.3). In contrast, the model with collateral constraints exhibits sizable propagation. This is displayed in Figure 3, which plots the response of key aggregate variables to the shock. The variable that exhibits the strongest reaction is the price of capital.[16] It initially drops by almost 5%, and it is still depressed by about 1% after four years. The interest rate increases initially by 1.8% and then stays moderately above the steady state level as the economy recovers.[17] Most importantly, the drop in output is sizable and persistent. It takes about six years for the economy to return roughly to the steady state. Next, I analyze the reasons behind the long lasting impact of one-time shocks.

Output depends not only on the amount of labor and capital in the economy, but also on the allocation of capital among entrepreneurs. Poor entrepreneurs operate at a less than efficient scale, because they get less than optimal financ-

[16]I choose to plot the price of new capital, q_t, rather than p_t. Result 5 states that $p_t = q_t - \delta$ if the capital stock is fixed.

[17]The reaction of the real interest rate is in line with data, where its contemporaneous correlation with output is small but positive (see, e.g., King and Watson 1996). Moreover, in a simple RBC model the interest rate would react similarly: anticipating higher productivity tomorrow, agents would like to dissave and thereby increase the equilibrium interest rate. In contrast, the real interest rate initially drops when a sufficiently persistent shock to TFP hits the economy, both in a simple RBC model and in this model (see Section 4.4).

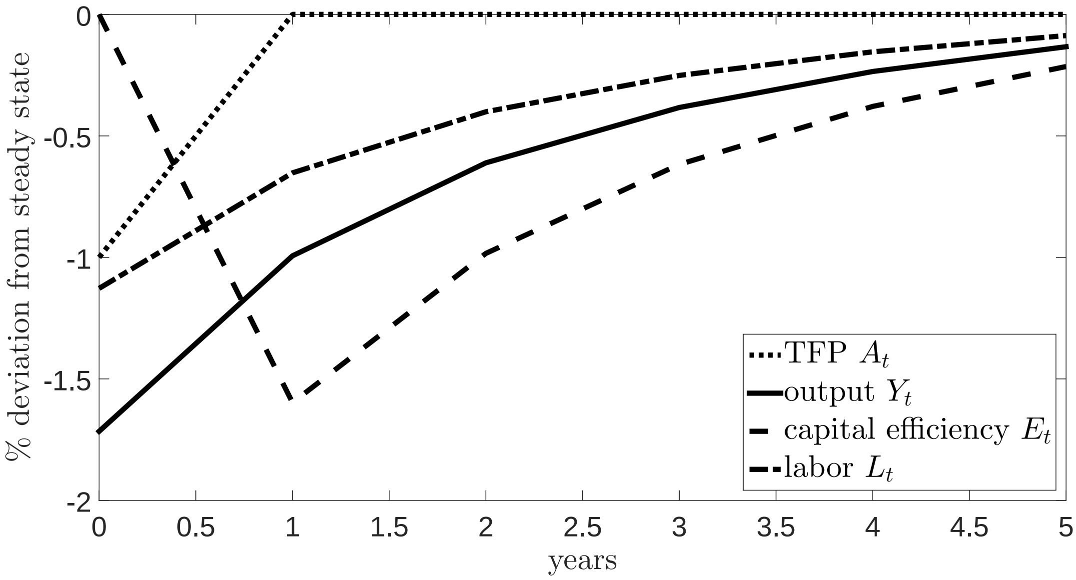

Figure 4: Decomposing the response of output.

ing. To quantify how (in)efficient capital is allocated among entrepreneurs, define *capital efficiency*,

$$E_t \equiv \frac{1}{K_t}\left(\int y_{i,t}^{\phi} di\right)^{1/\phi},$$

and observe how E_t enters aggregate output:

$$Y_t = A_t E_t^{\alpha} K_t^{\alpha} L_t^{1-\alpha}.$$

With $\tilde{X}$ denoting the log-deviation of the variable X from its steady state value, the output response may be decomposed as follows:

$$\tilde{Y}_t = \tilde{A}_t + \alpha\tilde{E}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{L}_t.$$

Remember that capital is fixed in the baseline calibration, thus $\tilde{K}_t = 0$ throughout. Note also that $\tilde{E}_0 = 0$, because the allocation of capital is determined one period ahead. Consequently, the initial drop in output is due only to the drop in TFP and the associated reaction in the labor supply. This drop would be of the same magnitude without financial frictions. However, from period $t = 1$ onward, output is diminished for a quite different reason, as can be seen from Figure 4. In period $t = 1$, TFP has already recovered, but capital efficiency is

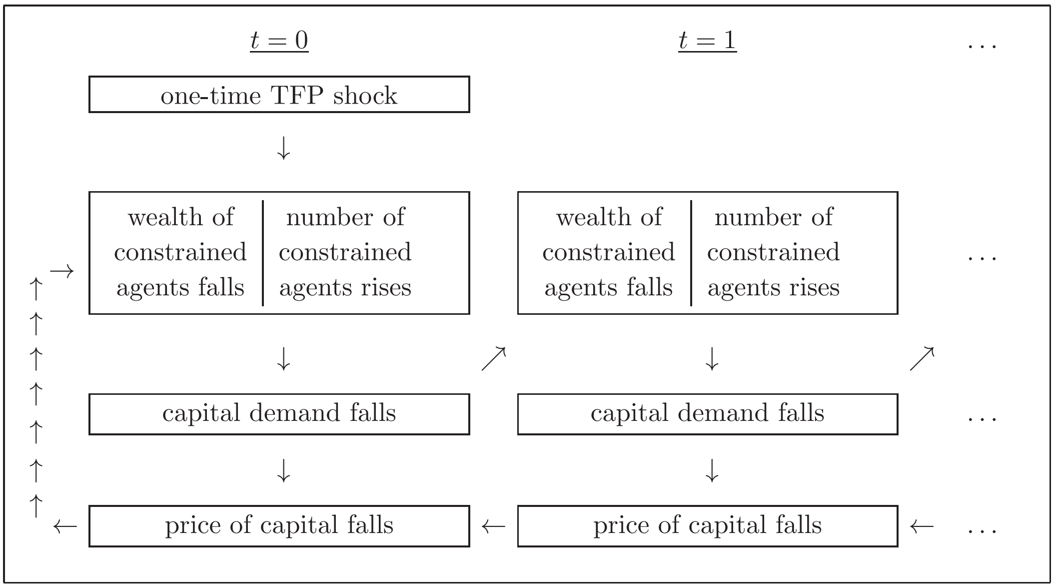

Figure 5: Collateral constraint mechanism.

now down by 1.6%. The direct effect of this is a reduction in output of about $1.6\% \times \alpha \approx 0.6\%$. On top of that, a lower capital efficiency implies a reduced marginal product of labor, which drags down labor supply by about 0.7% resulting in an additional drop in output of $0.7\% \times (1 - \alpha) \approx 0.4\%$. Summing up, a one-time negative shock to TFP causes a persistent drop in capital efficiency, which in turn depresses labor and output for several years.

The drop in capital efficiency is caused by the collateral constraints. The basic mechanism at work is illustrated in Figure 5. The key role is played by the constrained entrepreneurs, who have low financial wealth, are highly leveraged, and operate at high marginal returns. In contrast to Kiyotaki and Moore (1997) the aggregate shock does not only reduce the wealth of constrained agents but also increases their number.

Consider the intra-temporal effects first. In period $t = 0$, TFP is low, which implies low returns on entrepreneurial investment. However, the repayment obligations that entrepreneurs face remain unaffected. Thus, the financial wealth of highly leveraged agents is reduced sharply. For agents who are collateral constrained—and their number is rising due to the shock—lower financial wealth necessitates lower investment in capital. In the aggregate, the reduced capital demand from the constrained entrepreneurs has to be offset by increased capital demand from rich entrepreneurs. However, to make them invest in spite of their low marginal returns, the price of capital has to fall. But a falling price

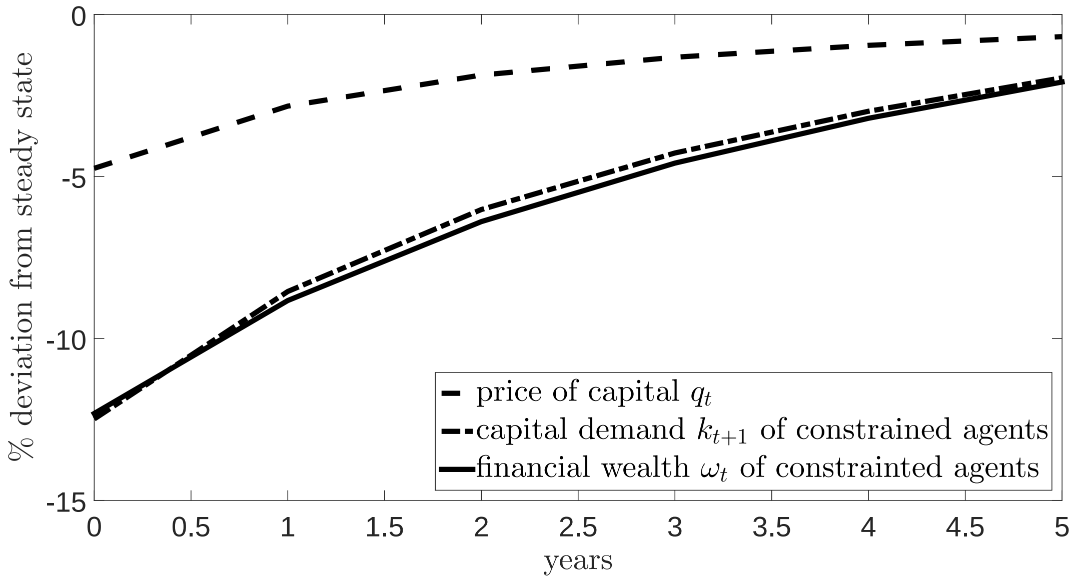

Figure 6: Quantifying the collateral constraint mechanism.

of capital further reduces the financial wealth of constrained entrepreneurs, which constitutes a powerful intra-temporal feedback effect. Now consider the inter-temporal effects. First of all, as constrained entrepreneurs have to forgo profitable investment opportunities in $t = 0$, they have less financial wealth in $t = 1$, which lowers the demand for capital and its price in that period. A lower price in $t = 1$ reduces the payoff to investments made in $t = 0$, thus further depressing the price in $t = 0$. In principle, these inter-temporal effects are effective up until $t = \infty$.

The quantitative significance of the collateral constraint mechanism is documented in Figure 6. It shows the response of constrained agents' financial wealth and capital demand. In period $t = 0$ their financial wealth falls by more than 12%, which causes a reduction of their capital holding of almost the same proportion. The reduced demand in turn causes the price of capital to drop by almost 3%, which accounts for most of the total loss in financial wealth of constrained agents. Thus, the intra-temporal feedback effect described above has quantitative bite. The fact that all three plotted variables remain depressed for several years indicates that the combined impact of the intra-temporal and the inter-temporal effects is substantial. This is not least because the number of constrained agents goes up as the shock hits. Figure 7 plots the resultant change to the percentage of constrained agents; it rises by more than 10% and

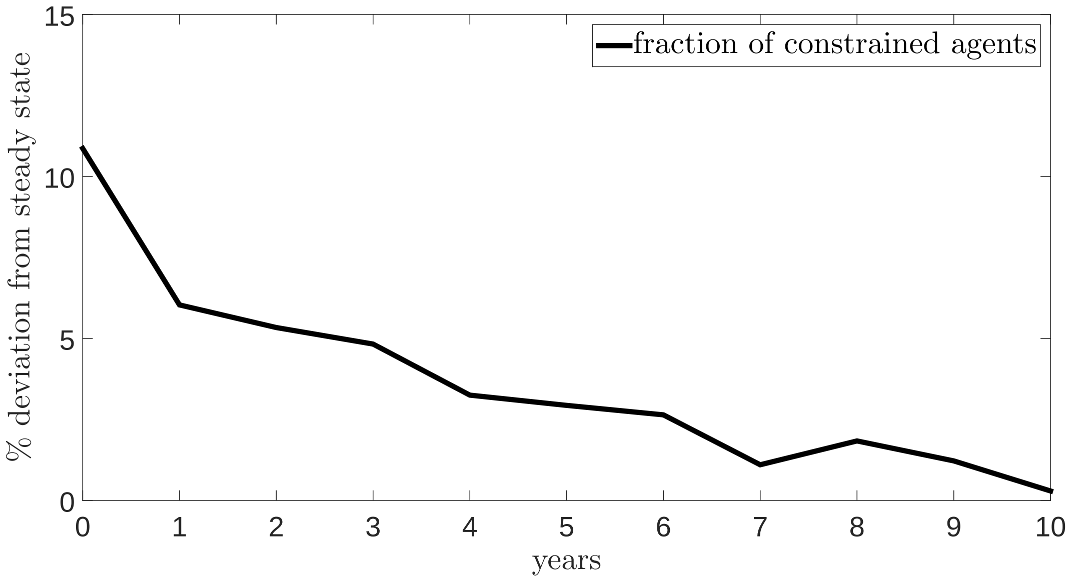

Figure 7: Response of the number of constrained entrepreneurs to negative shock.

takes about 10 years to return to its steady state level. This constitutes an extensive margin of the collateral constraint mechanism, which is not present in Kiyotaki and Moore (1997).

The feature that the number of constrained agents reacts to shocks has an interesting implication: the size of the response to a shock is a markedly non-linear function of its magnitude.[18] In particular, a negative shock drags down the economy by more than a positive shock of the same magnitude boosts the economy. This may help explain why recessions tend to be sharper than booms, as econometric studies like Hamilton (1989) find. To quantify the non-linearity, I define two measures for the response in output, which I call amplification and persistence. As collateral constraints have no impact on the reaction of output in $t = 0$, I follow Kocherlakota (2000) and measure amplification as the deviation of output in $t = 1$ relative to the shock in $t = 0$, thus amplification is

[18]Kocherlakota (2000) presents a knife-edge example of this non-linearity. In the steady state of his model, the collateral constraints of all agents are *just* binding. As a consequence, all agents are constrained and there is positive amplification in the case of a bad shock. In contrast, nobody is constrained in case of a good shock and there is no amplification. By including more heterogeneity among agents and using global solution techniques, I go beyond this artificial case.

Size of TFP Shock $\Delta \equiv \left(A_0 - \bar{A}\right)/\bar{A}$	Amplification $\left(\left(Y_1 - \bar{Y}\right)/\bar{Y}\right)/\Delta$	Persistence (Half-life)	Change of Constrained agents
+1.0%	0.81	1.5	−8.1%
−0.5%	0.92	1.5	+4.9%
−1.0%	0.99	1.5	+10.9%

Table 2: Impact of differently sized shocks

defined as

$$\left(\left(Y_1 - \bar{Y}\right)/\bar{Y}\right)/\Delta, \text{ where } \Delta \equiv \left(A_0 - \bar{A}\right)/\bar{A}.$$

If measured in this way, amplification is zero in the *complete markets benchmark*, as Appendix A.3 shows. Thus, any positive value signifies an impact of market incompleteness and collateral constraints. To disentangle those two effects, I also consider a *loose constraints benchmark* in which the collateral constraint is set so loosely that it is not binding for any agent (in particular, this requires $\kappa = 1.1$; see Table 3). In this case, amplification is about one-third as large, implying that two-thirds of the amplification in the baseline comes from tight collateral constraints while one-third comes from incomplete markets. As a measure of the persistence of the impact on output, I use the half-life of the $t = 1$ reaction to output—that is, I calculate how many years it takes until output is only depressed by half as much as in $t = 1$. A value of 1.5 thus means that two and a-half years after the shock, output is depressed half as much as 1 year after the shock. Both amplification and persistence would be equal across shocks if the response was a linear function of the impulse. This is the case for persistence, yet clearly not for amplification. For instance, there is 22% more amplification in case of a −1% shock compared to a +1% shock.
The last column in Table 2 reports how the number of entrepreneurs that are constrained changes through the shock, which suggests that the percentage of constrained agents is indeed a major driving force for the non-linearity in the relation between impulse and response.

4.3 Sensitivity Analysis

The results in Section 4.2 show that collateral constraints can generate large amplification and persistence. This subsection analyzes how sensitive these find-

Sensitivity w.r.t.	Value	Amplification	Persistence
Risk aversion γ	2	0.99	1.5
	1.5	0.85	1.3
Frisch elasticity $1/\theta$	1.9	0.99	1.5
	1	0.69	1.4
Adjustment costs ξ	$+\infty$	0.99	1.5
	10	0.59	2.0
	0	0.22	14.7
Collateralizability κ	0.76	0.99	1.5
	0.86	0.86	1.4
	1	0.64	0.9
	1.1	0.27	0.8
Persistence of labor income ρ	0.53	0.99	1.5
	0.95	0.95	1.5
Entrepreneurs' prod. a^e	2.63	0.99	1.5
	2.45, 2.81	1.02	1.5
Price entering CC p	p_t	0.99	1.5
	p_{t+1}	0.57	1.6

Table 3: Sensitivity analysis

ings are to the value of crucial parameters, namely to risk aversion, γ, the Frisch elasticity, $1/\theta$, the capital adjustment cost parameter, ξ, the collateralizability of capital, κ, and the persistence of workers' labor income, ρ. Furthermore, I consider higher variation in entrepreneurial productivity and a different collateral constraint. Overall, it turns out that some of these changes have a substantial impact on the dynamics of the model, but none reduces the impact of collateral constraints to a negligible size. This robustness stands in contrast to models with heterogeneous preferences, where—for example—Cordoba and Ripoll (2004b) find that substantial amplification is a knife-edge result only.

First, consider risk aversion. According to Table 3, higher risk aversion (and thus lower inter-temporal elasticity of substitution) leads to higher amplification *and* persistence. This is in line with Pintus (2011), who shows that the trade-off between amplification and persistence found by Kiyotaki and Moore (1997) does not necessarily arise in models with collateral constraints. Next, consider different values for the Frisch elasticity. To put them into perspective, note that in a meta-analysis of existing evidence Chetty et al. (2011) report a micro estimate of 0.82 and a macro estimate of 2.84. Clearly, a lower Frisch elasticity reduces amplification and persistence, as labor supply reacts less to changes in the efficiency of capital allocation. However, even with a Frisch elasticity of 1, there is still substantial amplification: 0.69% compared to 0% with complete markets. Adjustment costs also have a large impact on aggregate dynamics: When the respective parameter ξ is reduced, amplification goes down, but persistence goes up. Without adjustment costs, the price of capital does not move at all and the collateral constraint mechanism cannot work. Instead, investment reacts to shocks, which changes the capital stock. The impact on output that this change in the capital stock has is far less pronounced than the one generated by the collateral constraint mechanism in the case of fixed capital. However, it persists for longer. For intermediate values of ξ both effects are at work. For $\xi = 10$, which is close to the $\xi = 8.5$ assumed by Lorenzoni and Walentin (2007), amplification is still large yet much smaller than with fixed capital, while persistence is larger. Another crucial parameter of the model is the collateralizability of capital, κ. For $\kappa = 1$, amplification and persistence are lower than for $\kappa = 0.76$. This is qualitatively in line with the results of Mendicino (2012). The persistence of workers' labor income has only a very weak impact on the main results of the paper: Increasing ρ massively from its baseline value of 0.53 to 0.95 only marginally reduces amplification. When it comes to entrepreneurial productivity, it turns out that the simplifying assumption that this parameter is the same for all entrepreneurs does in fact bias the results against strong amplification. In a scenario with two types of entrepreneurs of equal number, one being 15% more productive than the other, there is slightly more amplification than in the baseline.[19] Finally, consider the alternative specification for the collateral constraint, presented in Section 2.3, which is $d_{t+1}R_{t+1} \leq \kappa p_{t+1}k_{t+1}$. For this specification amplification is 0.57 with a half-life of 1.6 years. Thus, persistence is now higher, yet amplification is smaller than in the baseline specification. The reason for this is as follows: As the shock is entirely non-persistent, the price of capital increases from period $t = 0$ to $t = 1$. Therefore, the collateral constraint is looser if it depends on next period's price rather than the current one. A looser constraint in turn leads to lower amplification. However, for different shock scenarios the price of capital might also go down for several periods. In such a case the alternative specification would generate stronger effects. An example of such a shock is an anticipated drop in future TFP or a sufficiently persistent drop in current TFP, which I consider in the next subsection.

[19] It is assumed that conditional on staying an entrepreneur, the probability of keeping the same productivity type (from one year to the next) is 90%.

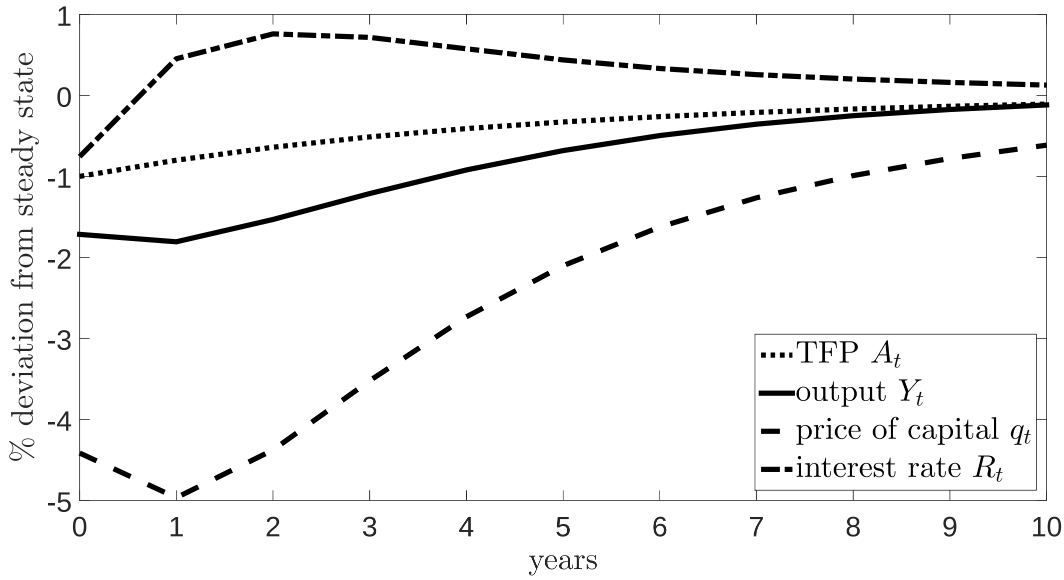

Figure 8: Response of output and prices to a persistent shock to TFP.

4.4 Response to a Persistent TFP Shock

Considering a one-time shock to TFP was helpful to clearly identify the amplification effect of collateral constraints. However, as a robustness check, I now consider the (arguably more realistic) scenario of a persistent drop in TFP. More precisely, the shock happens in period $t = 0$, where TFP drops by 1%; afterward, TFP does not immediately return to the steady state in period one, but only closes the gap to the steady state by 20% from one period to the next, as illustrated in Figure 8. This figure also plots the response of key aggregate variables to the shock. The variable that exhibits the strongest reaction to the shock is again the price of capital. It initially drops by almost 4.5% and falls to an even lower level in period 1. Output in period 1 now declines by 1.8% to a level that is also even lower than in period 0. In period 2, it is still depressed by 1.5%, compared to 1.1% with complete markets. Interestingly, the interest rate now initially falls slightly and then increases as the economy recovers. The deviations in both directions are small—namely, below 0.8%. This moderate reaction of the interest rate sharply distinguishes my model from some of the previous literature, the implications of which I will now discuss.

4.5 Comparison to the Previous Literature

While a changing number of constrained agents is a nice feature of this model, it cannot entirely explain why there is more propagation than in previous papers that consider calibrated general equilibrium models with collateral constraints, like that of Cordoba and Ripoll (2004b) or Mendicino (2012). Such models assume two types of entrepreneurs: impatient ones who are collateral constrained, and patient ones who lend money to the impatient. By assuming strongly heterogeneous preferences (e.g., quarterly discount rates of 0.99 versus 0.89 in Cordoba and Ripoll 2004a) these papers can generate very large differences in marginal productivity. Therefore, reallocation of capital can have substantial effects on output. However, these models do not create enough reallocation of capital in the first place, and to the extent that they do, reallocation is only short-lived, as Cordoba and Ripoll (2004b) show with the help of an extreme calibration. The main reason why reallocation of capital is feeble in these models is that *all borrowers* are constrained. As the shock reduces their financial wealth, demand for debt falls dramatically. This reduction in the demand for debt is not met by a corresponding reduction in its supply, as lenders are patient and still want to save. Thus, for bond markets to clear, the interest rate has to fall sharply.[20] This drop in the interest rate in turn mitigates the collateral constraint mechanism for two reasons. First, a lower interest rate loosens the collateral constraint, which implies less capital reallocation. Second, cheap loans increase the returns on leveraged investments in capital, which helps the wealth of constrained agents to recover quickly. Consequently, the reallocation of capital is short-lived. In contrast, capital reallocation is large and persistent in my model where *not all borrowers* are constrained. After a shock hits, unconstrained entrepreneurs take up additional debt and invest in capital in order to profit from the latter's expected rise in price. Therefore, the interest rate does not fall substantially (depending on the persistence of the shock it either slightly falls or slightly rises), and the collateral constraint mechanism is thus not mitigated. In addition, such moderate movements of the real interest rate are in line with its low volatility in the data (see, e.g., King and Watson 1996).

[20] See, for instance, Figure 7 in Cordoba and Ripoll (2004b), which plots the reaction to a 1% *increase* in productivity: a fall in the bond price of about 25%. This implies a dramatic rise in the interest rate—and a dramatic fall in the case of a negative shock. Note, however, that in calibrations with less heterogeneity in discount factors, e.g. in Mendicino (2012) and Punzi and Rabitsch (2015), the reaction of the interest rate, while qualitatively the same, is less dramatic.

5 Conclusion

The seminal work of Kiyotaki and Moore (1997) raised the question whether collateral constraints substantially amplify and propagate shocks to aggregate productivity, in particular whether they do so in less stylized models as well. Early answers to this question by Kocherlakota (2000) and Cordoba and Ripoll (2004b) were largely negative: they did not find robust amplification. Later studies found stronger effects of collateral constraints by considering different types of shocks (e.g. Khan and Thomas (2013)) or different types of constraints (e.g. Bassetto et al. (2015)). This paper reconsiders the original question, yet changes the force that makes collateral constraints (occasionally) binding. Instead of assuming heterogeneous patience, I follow the literature on entrepreneurship (e.g. Quadrini (2000)) and model idiosyncratic risk with respect to agents' productivity as workers and entrepreneurs. It turns out that within this framework shocks to aggregate productivity are strongly amplified and propagated over time. Moreover, I show in a comprehensive sensitivity analysis that this effect remains strong when crucial parameters of the model are changed.

In the baseline calibration, an unanticipated shock that reduces TFP by 1 percent for only 1 year has large and persistent effects: one year after the shock, output is still down by about 1 percent and the price of capital by almost 5 percent. These aggregate dynamics are quite persistent and are driven by a substantial reallocation of capital. When the shock hits, the wealth of constrained entrepreneurs falls by 11 percent, which forces them to reduce their capital holdings also by 11 percent. Conversely, unconstrained entrepreneurs increase their capital holdings to take advantage of the low prices. However, they operate at lower marginal returns, thus capital is used less efficiently, which in turn implies lower aggregate output. On top of that mechanism, the reaction to the shock is also driven by an increase in the number of constrained agents. This goes up by more than 10 percent on impact and only slowly recovers. Endogenous changes in the number of constrained agents have a further noteworthy implication: The response to a shock is far from being a linear function of its magnitude. If the size of a negative shock is doubled, the impact on output is much more than doubled; also, a negative shock depresses the economy by much more than a positive shock of the same size boosts it, thus recessions are sharper than booms.

A Appendix

A.1 Proof of Proposition 1

Result 1 (FOCs of the Individual Problem)
The FOCs of the individual optimization problem are for all $t \in \mathbb{N}$:

$$
\begin{aligned}
\textit{BC:} \quad & 0 = \pi_t k_t f(a^e_{t-1}, a^e_t) + p_t k_t - d_t R_t + w_t h_t a^w_t - c_t - q_t k_{t+1} + d_{t+1} \\
\textit{CC:} \quad & 0 \leq \kappa p_t k_{t+1} - d_{t+1} R_{t+t} \;\wedge\; 0 \leq \lambda_t \;\wedge\; 0 = (\kappa p_t k_{t+1} - d_{t+1} R_{t+1})\,\lambda_t \\
\textit{SC:} \quad & 0 \leq k_{t+1} \;\wedge\; 0 \leq \nu_t \;\wedge\; 0 = k_{t+1}\nu_t \\
k_{t+1}\textit{:} \quad & 0 = -u_c(c_t, h_t) q_t + \nu_t + \lambda_t \kappa p_t \\
& \quad + \beta \mathbb{E}\left[u_c(c_{t+1}, h_{t+1}) \left(\frac{\partial\,(\pi_{t+1} k_{t+1})}{\partial k_{t+1}} f(a^e_t, a^e_{t+1}) + p_{t+1} \right) \right] \\
d_{t+1}\textit{:} \quad & 0 = u_c(c_t, h_t) - \lambda_t R_{t+1} - \beta R_{t+1} \mathbb{E}\left[u_c(c_{t+1}, h_{t+1}) \right] \\
h_t\textit{:} \quad & 0 = u_h(c_t, h_t) + u_c(c_t, h_t) w a^w_t
\end{aligned}
$$

Proof:
Follows from differentiating the Lagrangian to the individual optimization problem and substituting the consumption Euler equation into the other equations. The variables λ and ν denote the multipliers for (CC) and (SC), respectively.□

Result 2 (Labor Supply and Wage)
$L_t = ((1-\alpha)\,Y_t)^{\frac{1}{1+\theta}}$, $\;w_t = ((1-\alpha)\,Y_t)^{\frac{\theta}{1+\theta}}$.

Proof:
From Result 1, the FOC with respect to h_t is:
$u_h(c_t, h_t) + u_c(c_t, h_t) w a^w_t = 0 \Leftrightarrow \left(c_t - \frac{h_t^{1+\theta}}{1+\theta} \right)^{-\gamma} (h^\theta_t - w a^w_t) = 0,$
which implies that optimal labor supply is given by $h_t = (w a^w_t)^{1/\theta}$. Consequently, aggregate efficiency units of labor are

$$L_t = \int a^w_{i,t} (w_t a^w_{i,t})^{1/\theta} di = w_t^{1/\theta} \int (a^w_{i,t})^{1+1/\theta} di = w_t^{1/\theta},$$

where the final step employs the normalization from Section 2.4. Using this and the FOC for final good production (which is $w_t = (1-\alpha)\,Y_t L_t^{-1}$) the above results follow. □

Result 3 (Price of Intermediate Goods)
The price of intermediate goods $y(x_t)$ is given by
$\pi(x_t) = Z y_t(x_t)^{\phi-1}$*, where* $Z \equiv \alpha\,(1-\alpha)^{\frac{(1-\alpha)\phi}{\alpha(1+\theta)}} A^{\phi/\alpha} Y^{\frac{\alpha-\phi}{\alpha} + \frac{(1-\alpha)\phi}{\alpha(1+\theta)}}$.

Proof:
The first order condition of the final output firm with respect to the input good from a firm with characteristics x_t provides

$$\begin{aligned}
\pi(x_t) &= \frac{\alpha A L^{1-\alpha}}{\phi}\left(\int y_t(x)^{\phi} d\Phi_t(x)\right)^{\frac{\alpha}{\phi}-1} \phi y_t(x_t)^{\phi-1} \\
&= \alpha A^{\phi/\alpha} Y^{\frac{\alpha-\phi}{\alpha}} L^{\frac{(1-\alpha)\phi}{\alpha}} y_t(x_t)^{\phi-1} \\
&= \alpha\left(1-\alpha\right)^{\frac{(1-\alpha)\phi}{\alpha(1+\theta)}} A^{\phi/\alpha} Y^{\frac{\alpha-\phi}{\alpha}+\frac{(1-\alpha)\phi}{\alpha(1+\theta)}} y_t(x_t)^{\phi-1} \\
&= Z y_t(x_t)^{\phi-1},
\end{aligned}$$

where Result 2 and the definition of Z are used. □

Result 4 (Price of Capital)
The prices of old and new capital are given by

$$p_t = (1-\delta) + \frac{\xi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2 - 1\right), \quad q_t = 1 + \xi\left(\frac{K_{t+1}}{K_t} - 1\right).$$

Proof:
As the price of 1 unit of investment is always equal to 1, the prices of old and new capital directly follow from differentiating

$$I_t(K_{t+1}, K_t) = K_{t+1} - (1-\delta)K_t + \frac{\xi}{2}\frac{\left(K_{t+1} - K_t\right)^2}{K_t}.$$

The price of old capital is given by the marginal reduction in investment due to a marginal increase of old capital, while the price of new capital is given by the marginal increase in investment needed to produce an additional marginal unit of new capital:

$$p_t = -\left(\frac{\partial I_t}{\partial K_t}\right), \quad q_t = \frac{\partial I_t}{\partial K_{t+1}}.$$

□

Result 5 (Price of Fixed Capital)
As adjustment costs ξ go to infinity, the prices of old and new capital satisfy

$$p_t = q_t - \delta.$$

Proof:
First use Result 4, then simplify, and finally take the limit:

$$\begin{aligned} p_t &= (1-\delta) + \frac{\xi}{2}\left(\left(\frac{q_t - 1}{\xi} + 1\right)^2 - 1\right) = q_t - \delta + \frac{(q_t-1)^2}{\xi}, \\ \lim_{\xi \to +\infty} p_t &= q_t - \delta + \lim_{\xi \to +\infty} \frac{(q_t-1)^2}{\xi} = q_t - \delta. \end{aligned}$$

□

A.2 Numerical Solution

Steady State

The numerical procedure used to solve for the steady state builds on the pioneering work of Aiyagari (1994). With a neoclassical production function and exogenous labor supply, the only aggregate variable that Aiyagari (1994) has to determine numerically is the interest rate—because output, capital stock, and the wage are all determined by the interest rate through the firm's FOCs. This is different in my model, where the distribution of capital among entrepreneurs matters for output. As a consequence, I have to compute both the interest rate and output, which is done by Algorithm 1. As this algorithm solves for a stationary equilibrium, there are no time indexes in its description.

Algorithm 1 (Solve for Steady State)

1. *Guess aggregate variables* $\{R, Y\}$.
2. *Given* $\{R, Y\}$, *solve for individual policy functions* $\{d, k\}$.
3. *Given* $\{R, Y\}$ *and* $\{d, k\}$, *find the stationary distribution* Φ.
4. *From* $\{d, k\}$ *and* Φ, *calculate implied net supply of debt and output* $\{\hat{D}, \hat{Y}\}$.
5. *Using* $\{\hat{D}, \hat{Y}\}$, *update the guess for* $\{R, Y\}$ *and go back to step 2.*

To understand why it is sufficient to determine the interest rate and output, consider the second step in Algorithm 1. In this step individual policy functions have to be computed given aggregate variables. From the FOCs of the individual problem (see Result 1) it is clear that the endogenous aggregate variables that

influence individual choice are $Y, p, q, R,$ and w. By Result 2, w follows from Y. From Result 4, it is obvious that the steady state prices of capital are simply $p_t = 1 - \delta, \; q_t = 1$. Hence, it indeed suffices to guess R and Y in order to compute individual policies.
Turning to the implementation of Algorithm 1, step 2 is carried out by iterating on policy functions. To reduce the number of continuous dimensions to the individual problem, I define financial wealth:

$$\omega_t \equiv \pi_t k_t f(a^e_{t-1}, a^e_t) + p_t k_t - d_t R_t.$$

To interpolate policy functions along this dimension, I use piece-wise linear interpolation. The grid of interpolation nodes is finer at lower levels of financial wealth and it is automatically refined near the kink induced by the collateral constraint (similar to in Brumm and Grill (2014)). To find the invariant distribution in step 3, I discretize the transition (between individual states (ω, a^w, a^e)), which is implied by $\{d, k\}$. For this purpose, I use a transition grid that is ten times finer than the interpolation grid, which results in a large transition matrix. Using standard numerical procedures, it is nevertheless possible to find the matrix's non-negative normalized eigenvector with eigenvalue one. If the transition grid is fine enough, this eigenvector provides a good approximation to the true invariant distribution over individual states (which is continuous). In step 4, the policies from step 2 are evaluated over the distribution from step 3 to get the implied level of output and net aggregate debt. Finally, step 5 employs a linear regression approach to update the guesses. Each iteration of Algorithm 1 generates deviations from equilibrium, $\{\hat{Y} - Y, \hat{D}\}$—the difference between the output level implied by k and Φ and the guess for output, as well as the deviation of the net supply of debt implied by d and Φ from zero net supply of debt. I regress the deviations from equilibrium, which I have from previous iterations of the algorithm, on the respective guesses used in these iterations. Then I use the coefficients of this regression to choose new guesses. These were determined such that there would be no deviations from equilibrium if the relation between guesses and deviations were linear. Clearly it is not linear; nevertheless the procedure converges very fast—it turned out to be much faster than a nested bisection method.

Transition Path

When it comes to the transition path, the computational burden increases substantially relative to the steady state for two reasons: First, as aggregate variables change along the transition path, sequences rather than steady state levels of aggregate variables have to be computed. Second, in addition to output and the interest rate, one additional aggregate variable has to be determined numerically, as the prices of capital are no longer determined by steady state conditions. I choose to guess the price of new capital, which implies the price of old capital and the evolution of the aggregate capital stock through Results 4 and 5 in Proposition 1. Accordingly, the transition path is computed as in Algorithm 2.

Algorithm 2 (Solve for the Transition Path)

1. *Choose a time horizon T and guess the transition path* $\{R_t, Y_t, q_t\}_{t\in\{0,\dots,T\}}$.

2. *Given* $\{R_t, Y_t, q_t\}_{t\in\{0,\dots,T\}}$, *solve backward for individual policy functions* $\{d_{t+1}, k_{t+1}\}_{t\in\{0,\dots,T\}}$.

3. *Given* $\{R_t, Y_t, q_t\}_{t\in\{0,\dots,T\}}$ *and* $\{d_{t+1}, k_{t+1}\}_{t\in\{0,\dots,T\}}$, *solve forward for the distributions* $\{\Phi_t\}_{t\in\{0,\dots,T\}}$.

4. *From* $\{d_{t+1}, k_{t+1}\}_{t\in\{0,\dots,T\}}$ *and* $\{\Phi_t\}_{t\in\{0,\dots,T\}}$, *calculate the implied transition path* $\{\hat{D}_t, \hat{Y}_t, \hat{K}_t\}_{t\in\{1,\dots,T\}}$.

5. *Using* $\{\hat{D}_t, \hat{Y}_t, \hat{K}_t\}_{t\in\{1,\dots,T\}}$, *update the guess for* $\{R_t, Y_t, q_t\}_{t\in\{0,\dots,T\}}$ *and go back to step 2. The update is determined as follows, where n denotes iterations of the algorithm, and χ is a decreasing function:*

$$
\begin{aligned}
R_{t,n+1} &= R_{t,n} + \hat{D}_{t,n} v_n, \\
Y_{t,n+1} &= Y_{t,n} + \left(\hat{Y}_{t,n} - Y_{t,n}\right) v_n \bar{Y}, \\
q_{t,n+1} &= q_{t,n} + \left(\hat{K}_{t,n} - K_{t,n}\right) v_n, \\
v_{n+1} &= v_n \cdot \chi\left(\min_t \left\{\min\left\{\frac{\hat{D}_{t,n}}{\hat{D}_{t,n-1}}, \frac{\hat{K}_{t,n} - K_{t,n}}{\hat{K}_{t,n-1} - K_{t,n-1}}, \frac{\hat{Y}_{t,n} - Y_{t,n}}{\hat{Y}_{t,n-1} - Y_{t,n-1}}\right\}\right\}\right), \\
K_{t,n} &= \begin{cases} \bar{K} & \text{if } \xi = +\infty \text{ or } t = 0 \\ \left(\frac{q_{t,n}-1}{\xi} + 1\right) K_{t-1,n} & \text{if } \xi < +\infty \text{ and } t > 0. \end{cases}
\end{aligned}
$$

Concerning step 1, T has to be chosen such that the economy indeed converges to the steady state within T years (up to the desired numerical precision). If this is not the case, T has to be increased. For steps 2, 3, and 4, the procedures used are similar to the ones used in Algorithm 1. It is step 5 that causes most problems. The guess for each of the $T \times 3$ variables (e.g., q_t) influences not only the implied value for the corresponding variable (e.g., $\hat{K}_t$), but also other concurrent variables (e.g., $\hat{D}_t$), future variables (e.g., $\hat{Y}_{t+1}$), and past variables (e.g., $\hat{q}_{t-1}$). Because of this, it is difficult to update the $T \times 3$ guesses in a way that makes the algorithm converge. The procedure I use to achieve this obeys the following principles: First, the new guess is given by the old guess plus a measure of the error in the old guess. For instance, as a measure of the error in the interest rate the implied aggregate net supply of debt is used. Second, the size of the update step is governed by an update factor, v_n, which is equal across all $T \times 3$ variables (except that it is scaled up by $\bar{Y}$ in case of output). Third, the update factor is itself updated depending on the speed of convergence. The relevant speed is the one of the variable which converges fastest. If this speed is very low, then the update-factor is increased. If it is too high, then the update-factor is reduced in order to avoid oscillating behavior. The update procedure implied by these three principles is stated in step 5 of Algorithm 2.

A.3 Complete Markets Benchmark

This Appendix analyzes a complete markets version of the model presented in Section 2. It verifies, given the calibration from Section 3, the statements made in Sections 3 and 4.2 about the relation between the two versions of the model. Suppose that markets are complete. Thus, agents may write (and enforce) contracts that are contingent on individual entrepreneurial output. As there is no aggregate risk, the expected marginal return on all assets is equalized in equilibrium. Consequently, all entrepreneurs operate at the same expected marginal return. In the baseline calibration with only one type of entrepreneur, this implies that all agents who are entrepreneurs in $t-1$ invest $K_t/\mu(s^e)$ units of capital. Among these agents only the ones who still have positive entrepreneurial productivity in period t are productive. The measure of these agents is $\mu(s^e)M(s^e, s^e)$. Thus, aggregate output is given by

$$Y_t = A_t \left(\int y_{i,t}^{\phi} di \right)^{\alpha/\phi} L_t^{1-\alpha} = A_t \left(\mu(s^e) M(s^e, s^e) (a_h^e)^{\phi} \left(\frac{K_t}{\mu(s^e)} \right)^{\phi} \right)^{\frac{\alpha}{\phi}} L_t^{1-\alpha}.$$

Finally, using the normalization from Section 3, which is

$$a_h^e = M(s^e, s^e)^{-1/\phi} \mu(s^e)^{(\phi-1)/\phi},$$

the aggregate production function turns out to be as claimed in Section 3:

$$Y_t = A_t \left(\mu(s^e) M(s^e, s^e) M(s^e, s^e)^{-1} \mu(s^e)^{(\phi-1)} \left(\frac{K_t}{\mu(s^e)} \right)^{\phi} \right)^{\frac{\alpha}{\phi}} L_t^{1-\alpha} = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

Building on this result, it is straightforward to analyze the transition path for the complete markets economy. Note that TFP and capital (which is assumed to be fixed) are both at their steady state levels from period $t = 1$ onward:

$$A_t = \bar{A}, K_t = \bar{K} \;\; \forall t \geq 1.$$

Using the aggregate production function just derived, it follows that

$$Y_t = \bar{A}\bar{K}^{\alpha} L_t^{1-\alpha} \;\; \forall t > 1.$$

In addition, Result 2 implies

$$L_t = ((1-\alpha)\, Y_t)^{\frac{1}{1+\theta}} \;\; \forall t.$$

Combined, these equations imply that $\forall t > 1 : Y_t = \bar{Y}, L_t = \bar{L}$. Thus, the economy returns to the steady state right after being hit by the shock, which verifies the claim made in Section 4.2.

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